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THE COMPETITIVE  
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PUBLIC SERVICES

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# The choice between the in-house provision and the competitive out-house provision of public services

by

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## **Abstract**

This study focuses on the production welfare services which are purchased by the government. We consider whether the delivery of the service should be public or private. The social impact of the service is determined by the satisfaction of unforeseen individual needs. Assuming that the information about these needs is ex post observed only by the producer of the service favours public production. Auctioning the right-to-produce contract, however, selects the most cost-efficient private supplier. Whether private production is optimal resolves from the trade-off between cost efficiency and a failure in meeting individual needs. We also analyse investments in monitoring.

Keyword: Private or public, Right-to-produce auction, Service production

JEL Classifications: L33, L80

## 1 Introduction

The public provision of not only public but also private goods is extensive in many developed countries. This especially concerns educational, health and social services. It seems that public provision easily leads to public production. But the extent to which public units actually produce the services which they provide varies a lot from one country to another. In the Nordic countries of Europe the public sector itself produces services to a much larger extent than in other OECD countries. It is also remarkable that public employment is concentrated in the production of private services rather than private goods, and more or less in that service segment in which the skill and educational requirements are above the average.

This study considers the fairly common mechanism in which the public provider arranges a competitive tender for the production of a large aggregate of publicly financed services. The service in the situation under consideration is defined to mean the responsibility of taking care of the needs of many people in a certain area and in a specific field. A service considered is then running foreign or security policy, running a hospital, running a school or running a home for the elderly. In our set-up we are able to analyse also whether the public provision of such private goods as educational, health or social services should be - or tends to be - in-house or private.

The literature on the public unit's role as a producer is rather scarce. The main strand of the literature considers the trade-off between market failure and governmental failure. Market failure is seen to arise as a consequence of informational incompleteness. As Hart (1995) emphasizes the difficulties in observing verifiably the variables which affect the final

outcome, the unforeseen nature contingencies and the relation-specific investments which easily lead to a hold-up problem generate losses for the provider of the service (see Hart 1995). Hart et al. (1997) consider the choice between in-house provision and outsourcing of a publicly provided service. In their model governmental failure appears as insufficient incentives to advance productivity.

Analysing the choice of a public provider either to produce itself or to outsource through competition, this study also stresses the central role of informational incompleteness. We, however, see both governmental and market failure somewhat differently from the previous research. We do consider that governmental failure accrues from a public entity's minimal incentives to exert effort as in Schmidt (1996), Hart et al. (1997) and Hart (2002). This study, instead, considers the lack of competition as being the main disadvantage of in-house provision. Modelling market failure, we see that it is rather the unobserved or unverifiably observed effectiveness of the service than unverifiable investments in cost saving and service quality which is the source of informational imperfection. Effectiveness is defined to describe the impact of a unit increase of an output on the maximized welfare. Effectiveness is then contingent not only on the quality but also on the eligibility of the actions to meet individual needs.

We stress that to produce this provision requires on-line information of customers' needs - which arise randomly and in an unforeseen way - and satisfaction of which determines effectiveness. For example, a measure to cure a patient can be of high quality but its impact on welfare can still be impaired by the fact that the measure concerned is not the most appropriate. The high variance of parametrized effectiveness is typical of the very services considered as distinction of many technical services and commodities.

It is also fairly normal that the producer must respond to arisen needs immediately, and so we assume that the contract between the provider and the producer must be signed before actual effectiveness materialises. Stressing the unforeseeable nature of the individual needs - whose satisfaction defines the social impact of the service - is recognised in the literature (see, for example, Sandmo, 2002) but its implications are not profoundly analysed.

In the basic model we also assume that information about the value of effectiveness is the producer's private information. It is then asymmetric information and not moral hazard (as in Hart et al., 1997) which, in the outsourcing, leads to contractual failure and restricts the provider's welfare. We think that both private and public producers have an equal opportunity and ability to observe the needs which arise, react to them and evaluate how a response to them affects social welfare. On the other hand, neither a public nor a private firm - which are run by a manager - is assumed to have any incentive to deviate from the owner's will. From this it follows that the privately owned firm tends to minimise the level of output (or quality) and deviate from the effectiveness norm, whereas a public firm is benevolent and pursues social welfare. To encounter the unresponsiveness related to the effectiveness parameter the public purchaser can only tend to control ex ante outputs in private production and, in this way, fight against the tendency towards underproduction (allocative losses).

The producers' marginal costs - which define a producer's cost efficiency - differ from each other and information about marginal costs is private. When the public provider outsources production, a private producer is selected by auctioning the right-to-produce

contract.<sup>1</sup> It then becomes evident that a private producer is more cost-efficient than an alternative public supplier, in particular, in a thick market with many potential producers. This result applies because the cost efficiency of a public producer is drawn randomly from the same distribution as the cost efficiency of each private producer who participates in the auction. According to this, in-house provision without an auction excludes the opportunity to pick the most cost efficient producer through an auction or some other competitive mechanism. For the government the most effective model to produce then resolves from the derived trade-off between allocative efficiency - which describes the success in satisfying arisen needs - and cost efficiency. Focusing on this trade-off, we analyse a situation in which a public auctioneer announces a reserve price for the producer's marginal costs above which the bids are rejected and the public purchaser itself produces the service. We then consider the factors which have impact on the expected scope of out-house or in-house provision. It is not surprising that the probability of private production increases with the number of producers and, respectively, decreases with the variance of the parameter which defines the effectiveness of individual outputs.

We extend our framework by allowing the public provider to invest in monitoring by which the public provider may become able to observe the effectiveness parameter in the out-house provision. Mutual observation of effectiveness leads to bargaining over the spilt of the additional benefit which accrues when output is adjusted according to the effectiveness parameter. The approach introduced to assess the public purchaser's

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<sup>1</sup>Owing to incomplete information about effectiveness, the auction mechanism cannot induce the private producer to choose a socially efficient production level as in Sappington and Stiglitz (1987).

incentives to invest in monitoring<sup>2</sup> is fairly new in the literature. We show that the public purchaser pursues social efficiency by investing in monitoring if his/her ex-post bargaining power is high enough. Rather few studies have considered the regulator's incentives to monitor the service quality and the implications of the monitoring institution on the outsourcing decision. Sørensen (2004) allows the regulator to invest in monitoring and shows in his pure moral hazard framework that an increase in monitoring can induce the worker to strive harder for service quality.

Our approach put an emphasis on competition and more or less abstracts from the hold-up problem which is central in the previous literature which focuses on public versus private provision. These studies consider a model in which production is divided into two phases: first, an investment in a fixed asset and, after that, the asset is used to provide the service.<sup>3</sup> The hold-up problem plays a central role owing to the sunk investments in the relation-specific asset, which is then later used in the delivery of the service. In Hart et al. (1997) and Hart (2002) the investments in a fixed asset, which are seen to improve quality and also reduce costs, precede the efforts to reduce costs at the expense of the service quality. In any case, these two types of actions raise a trade-off between cost savings and quality improvements. This dualism in the effort setting also characterises Sørensen's (2004) framework, in which the agent can promote either his/her private interests or,

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<sup>2</sup>Previously, Williamson (1964), Mirrlees (1975) and Calvo and Wellisz (1978) have recognised that the public supervisors must be given sufficient incentives to exert supervisory effort.

<sup>3</sup>In this framework it is not only the issue between public ownership and provision versus private ownership and provision (as in Schmidt (1996) and in Hart et al. (1997)) but one can also consider whether the government should own the asset and purchase the service from a private provider or whether the ownership should also be allocated to the private firm (as in Bentz et al., 2003).

alternatively, exert effort which improves the quality of the service. Insofar as the outcomes are unverifiable, the split of benefits can only be based on incomplete contracting. In the model by Hart et al. (1997) the extent to which the government (the purchaser) "holds up" the producer in the bargain over the mutually observed surplus is contingent on the ownership of the non-human assets. Private ownership also creates stronger incentives to exert effort to save costs and deteriorate quality. At best, the quality deterioration associated with private production remains minor. In Schmidt's (1996) adverse selection framework, private ownership makes it possible for the governmental purchaser to commit to hard budget constraints. In its strivings toward costs savings, the private firm then outperforms the public firm which faces a soft budget constraint. But performance in private ownership is impaired by the allocative inefficiency owing to private information about cost parameters.

In our study we take for granted the fact that the service under consideration is publicly financed and do not address the issue as to whether the provision of the service should be public or private. The role of the public provision of private goods is increasingly explored in the literature. One explanation for the public sector's active role in the provision considers public provision as a device to alleviate the self-selection constraint which restricts redistribution that can be accomplished by an optimal non-linear income tax (see Cremer and Gahvari (1997) and Blomquist and Christiansen (1999)). Another explanation which analyses voting models sees public provision as a means to capture extra benefits at the expense of others (see Epple and Romano, 1998). Some studies in the optimal tax literature have also focused on public production and have regarded the redistributive role of public employment as the main motivation for public production.

Through its intervention in production and the labour market the government can favour skilled people and, in this way, encourage people to invest in education (Wilson, 1982) or, it can favour the low-skilled and, in this way, reduce wage differentials, which for its part allows to alleviate the redistributive burden of income tax (Pirttilä and Tuomala, 2002).

The rest of this study is organised as follows: In Chapter 2 we introduce the model. In Chapter 3 we consider the results and compare the benefits of public production and private production from the public purchaser's viewpoint. In Chapter 4 we extend the analysis to cover monitoring investments. Finally in Chapter 5 we present conclusions.

## 2 The model

In the model considered the purchaser of the service is always public. But the producer can be either a public body or a private firm. In the real world, the contract - which defines the provision of welfare services - covers a wide range of different outputs and needs which should be satisfied. However, without the loss of generality, we consider the provision of a single output. The value of a service for the provider is assumed to be  $\beta S(q)$  so that  $S' > 0$ ,  $S'' < 0$  and  $q$  is the individual output. The effectiveness parameter  $\beta$  denotes the value of  $q$  for the public purchaser. We assume that  $\beta$  is uniformly distributed in the range  $[\underline{\beta}, \bar{\beta}]$ .  $G(\beta)$  is  $\beta$ 's distribution function and  $G'(\beta)$  is its density function. The value of  $\beta$  resolves when the need concerning  $q$  arises.

The production costs are  $\theta_k q$  and so linear in  $q$ .  $\theta_k$  is then a producer  $k$ 's marginal cost. The potential private producers are ex ante symmetric. Nature selects each producer's type randomly from a common distribution  $F(\theta_k)$ , which has a support  $[\underline{\theta}, \bar{\theta}]$ . The marginal costs  $\theta_k$  is a producer  $k$ 's private information. The number of potential private

producers is  $n$ . A parameter  $\theta_g$  denotes a public body's efficiency parameter, which is also drawn randomly from  $F(\theta_k)$ .

In the basic model, the effectiveness parameter  $\beta$  is observed only by a producer, and so in private production it is observed only by the winning supplier after the auction. Delegating the production to the private firm, the public authority loses its ability to see the needs which arise randomly, and so the public purchaser does not observe  $\beta$  when it materialises under private production. While it carries out the production itself, the public body observes  $\beta$  and can react to it. The output  $q$ , instead, is verifiably observed.

In private production the public purchaser sells the right-to-produce contract in a sealed, high-bid auction, which is analysed more closely in Dasgupta and Spulber (1989/90) and in Chen (2004). In the right-to-produce auction the auctioneer specifies the quantity-payment schedule  $P(q)$ , which defines how much the purchaser pays for an output  $q$ . Knowing  $P(\cdot)$ , the suppliers each name a quantity in a sealed bid. The suppliers make bids noncooperatively and we will also show that their bidding follows a common equilibrium strategy  $q(\theta_k)$  so that  $x_k = \theta_k$  when  $x_k$  is the marginal cost insisted upon on which firm  $k$ 's bidding is based. Then a supplier whose marginal cost is  $\theta_k$  bids  $q(\theta_k)$  and for a bidder it is not worth deviating from  $q(\theta_k)$  when this strategy is adopted by the other bidders. We also allow the purchaser to discourage inefficient bidders to enter the auction. According to this, the purchaser cuts the quantity-payment schedule so that  $P(q(x_k)) = 0$  if  $q(x_k) < q(\theta^o)$  when  $q(x_k)$  is the type  $k$ 's bid. The reserve price  $\theta^o$  implicitly defines the reservation level for  $\theta_k$ , above which it does not pay for the purchaser to induce the private producer to make bids.

In an extended version of the model we allow the public purchaser to become able to

monitor unverifiable  $\beta$  after investing in monitoring capacity. We then assume that with a given probability  $p(h)$  the purchaser is able to observe  $\beta$  when  $h$  denotes the monitoring effort which is verifiably observed. Then  $p(0) = 0$  and  $p'(h) > 0$ ,  $p''(h) < 0$  when  $h > 0$ . If in private production the public purchaser also observes unverifiable  $\beta$  after the producer is selected,  $q$  can be adjusted to correspond to actual  $\beta$  and not only to  $E(\beta)$  as in the absence of monitoring. This creates an additional benefit and the parties will bargain about how to split this gain. Contractual agreements about the readjustment of  $q$  according to mutually observed  $\beta$  are ruled out, because  $\beta$  is not verifiable. The purchaser is assumed to obtain a share  $\lambda$ , ( $0 \leq \lambda \leq 1$ ) of the additional gain. The level of  $\lambda$  resembles the purchaser's ex post bargaining power.

We also make two additional assumptions.

**Assumption 1.**  $q^{**}$ , which is defined by the condition  $\underline{\beta}S'(q) - \bar{\theta} = 0$ , is positive.

This assumption guarantees that the interior solution for  $q$  and the purchaser's payoff are positive in public production and that the system-wide payoff is also positive in all cases. We also introduce another assumption.

**Assumption 2.**  $q^*$  defined by  $E(\beta)S'(q) - (2\theta_k - \underline{\theta}) = 0$  is positive when  $\theta_k < \bar{\theta}$  and non-negative when  $\theta_k = \bar{\theta}$ .

Assumption 2 guarantees that in the private production (in the basic model) all the suppliers make positive bids insofar as  $q(x_k) > q(\theta^o)$ .

**Assumption 3.**  $x_k + \frac{F(x_k)}{F'(x_k)}$  increases in  $x_k$ .

Assumption 3 guarantees that the auction considered is regular (see Myerson (1981) and Chen (2004)).

### 3 Results from the basic model

In this chapter we discuss the implications of the right-to-produce auction. The order of events is the following: Nature selects  $\theta_k$  ( $k = 1, \dots, n$ ) and  $\theta_g$ . The purchaser then defines the quantity-payment schedule  $P(q)$  and sets the reserve price  $\theta^o$ , after which the purchaser arranges a sealed high-bid auction in which the right to produce is sold to the firm which bids the maximum quantity insofar as  $q(\theta_k) > q(\theta^o)$ . Next, the winning firm or the public provider itself produces the service during which the effectiveness parameter  $\beta$  materialises. In the case in which the auction selects a private producer, he is paid according to  $P(q)$ , if the quantity target specified is met. Otherwise, the producer is punished. We next consider in-house provision (i.e. pure public production) more closely and after that we evaluate the implications of contracting out through an auction.

#### 3.1 Public production

In public production the production phase succeeds the resolution of  $\theta_g$  and the output decision is adjusted according to an observed  $\beta$ . The purchaser's expected payoff with respect to random  $\beta$  in public production,  $E_\beta(WG(\theta_g))$ , has an expression  $\int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q) - \theta_g q] G'(\beta) d\beta$ . Optimal  $q$ , denoted  $q^{**}$ , is obtained from  $\arg \max_q WG$ . The assumptions of the model guarantee that the first-order conditions

$$\beta S'(q) - \theta_g = 0 \tag{1}$$

define  $q^{**}$ .  $E_\beta(\widehat{WG})$  - which denotes the purchaser's expected payoff in the maximum in public production - then has the equation

$$E_\beta(\widehat{WG}(\theta_g)) = \int_{\underline{\beta}}^{\bar{\beta}} (\beta S(q^{**}) - \theta_g q^{**}) G'(\beta) d\beta. \tag{2}$$

### 3.2 Private provision and auctioning supply contracts.

Taking into account that the output is set independently of  $\beta$ , the expected system-wide payoff from private production is

$$E_{\theta}^{\theta^o}(SWP1(\theta_k)) := \int_{\underline{\theta}}^{\theta^o} [E(\beta)(S(q(x_k)) - x_k q(x_k))] f_{(1)}(x_k) dx_k \quad (3)$$

where  $f_{(1)}(x_k) = nF'(x_k)(1-F(x_k))^{n-1}$  is the density function of  $\Phi \equiv \min\{\theta_1, \theta_2, \dots, \theta_n\}$  and  $q(x_k)$  represents firm  $k$ 's bid. In the auction considered the purchaser pays  $P(q(x_k))$  for  $q(x_k)$  units of output when  $q(x_k) \geq q(\theta^o)$ . We then obtain for supplier  $k$ 's profits

$$\Pi^k(\theta_k, x_k) = [P(q(x_k)) - \theta_k q(x_k)](1 - F(x_k))^{n-1} \quad (4)$$

when  $q(x_k) \geq q(\theta^o)$  and  $(1 - F(x_k))^{n-1}$  is the probability of winning. When  $q(x_k) < q(\theta^o)$  the supplier does not bid and the profits are zero. The truth-telling condition which makes firm  $k$  bid according to  $\theta_k$  is then

$$\left. \frac{\partial \Pi^k(\theta_k, x_k)}{\partial x_k} \right|_{x_k=\theta_k} = 0.$$

for any  $\theta_k \in [\underline{\theta}, \theta^o]$ . This condition implies that

$$\left. \frac{\partial \Pi^k(\theta_k, x_k)}{\partial \theta_k} \right|_{x_k=\theta_k} = -q(x_k)(1 - F(x_k))^{n-1}. \quad (5)$$

The second order condition that  $\left. \frac{\partial \Pi^k(\theta_k, x_k)}{\partial x_k} \right|_{x_k=\theta_k} = 0$  defines a maximum so that the incentive compatibility constraint  $\Pi_b^k(\theta_k, \theta_k) \geq \Pi_b^k(\theta_k, x_k)$  is fulfilled, requires that  $\frac{\partial^2 \Pi^k(\theta_k, x_k)}{\partial x_k \partial \theta_k} \geq 0$ . We obtain from (5)

$$\frac{\partial^2 \Pi^k(\theta_k, x_k)}{\partial x_k \partial \theta_k} = -\frac{\partial q(x_k)}{\partial x_k} (1 - F(x_k))^{n-1} + q(x_k)(n-1)(1 - F(x_k))^{n-2} F'(x_k) \quad (6)$$

which is non-negative because  $\frac{\partial q(x_k)}{\partial x_k}$ , derived from the first-order condition (11) below, is negative. Let  $\Pi_{qP}^k$  denote  $\frac{\partial(\Pi^k(\theta_k, x_k)/\partial q)}{\partial(\Pi^k(\theta_k, x_k)/\partial P)}$ , the marginal rate of substitution. The sufficient

condition that the auction implements the optimal strategy  $q^*(x_k)$  (defined in (11)) is also valid because the model considered meets the so-called Mirrlees-Spence property or single-crossing property (see Guesnerie and Laffont (1984) and Laffont and Martimort (2002)), according to which  $\frac{\partial \Pi_{qP}^k}{\partial \theta_k} = -1$  preserves its sign.<sup>4</sup>

The winning supplier's willingness to pay for the right to produce is not a function of parameter  $\beta$ , and so no such efficiency-improving mechanism exists which would implement the symmetric-information allocation in which output is set on the basis of the true value of the effectiveness parameter. We can say that firm  $k$  is unresponsive with respect to  $\beta$ <sup>5</sup>. This excludes an opportunity to renegotiate the contract in private production after the real value of the effectiveness parameter is exposed to the producer.

Define  $\Pi^k(\theta_k) = \Pi^k(\theta_k, \theta_k)$ . Setting  $\Pi^k(\theta^o) = 0$ , one then obtains from condition (5) for the producer's profits

$$\Pi^k(\theta_k) = \int_{\theta_k}^{\theta^o} q(x_k)(1 - F(x_k))^{n-1} dx_k. \quad (7)$$

Equating the right-hand side of (7) to the right-hand side of  $\Pi^k(\theta_k, \theta_k)$  in (4), one obtains the expression

$$P(q(\theta_k)) = \theta_k q(\theta_k) + \frac{\int_{\theta_k}^{\theta^o} q(x_k)(1 - F(x_k))^{n-1} dx_k}{(1 - F(\theta_k))^{n-1}} \quad (8)$$

for a payment function for all  $\theta_k \in [\underline{\theta}, \theta^o]$ . The latter term on the right-hand side of

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<sup>4</sup>The boundary condition, being a part of sufficient conditions, is also valid in the situation considered (see more closely Guenerie and Laffont (1984)).

<sup>5</sup>Caillaud et al. (1988) refer by unresponsiveness to the situation, analysed by Guenerie and Laffont (1984), in which the implementation of incentive-compatible allocation would conflict with the principal's interests. Implementability - when an agent possesses private information - is also considered more closely in Laffont and Martimort (2002).

(8) reflects the size of an informational rent which must be paid to deter each bidder from mimicking less efficient bidders and to induce each bidder to reveal their true marginal cost in the auction.

The expected profits,  $E_{\theta}^{\theta^o}(\Pi^k(\theta_k))$ , derived from (7) are then (see Chen, 2004)

$$E_{\theta}^{\theta^o}(\Pi^k(\theta_k)) = \int_{\underline{\theta}}^{\theta^o} q(x_k)(1 - F(x_k))^{n-1} F(x_k) dx_k. \quad (9)$$

The sum of the producers' expected profits is  $n$  times the expression (9). The purchaser's expected profits from private production are obtained by subtracting  $nE_{\theta}^{\theta^o}(\Pi^k(x_k))$  from  $E_{\theta}^{\theta^o}(SWP1(x_k))$  in (3). This difference can then be written as

$$E_{\theta}^{\theta^o}(WP(\theta_k)) := \int_{\underline{\theta}}^{\theta^o} [E(\beta)S(q(x_k)) - (x_k + \frac{F(x_k)}{F'(x_k)})q(x_k)]f_{(1)}(x_k)dx_k. \quad (10)$$

In (10) it has also been taken into account that the government chooses the optimal  $q$  independently of actual  $\beta$ . Let  $q^*(x_k) \equiv \arg \max WP(x_k)$ . The condition

$$E(\beta)S'(q) - (x_k + \frac{F(x_k)}{F'(x_k)}) = 0. \quad (11)$$

then defines  $q^*(x_k)$ . The schedule  $q^*(x_k)$  then maximizes the purchaser's expected profits (10), because the necessary second order condition (6) guarantees that  $q^*(x_k)$  arises as a Bayesian-Nash equilibrium in the auction considered. Consequently  $\Pi^k(\theta_k, x_k)$  is maximized at  $x_k = \theta_k$ .

After the winning supplier has been selected, however, an opportunity arises for both the purchaser and the private supplier to gain some additional benefit by adjusting the product plan from  $q^*(x_k)$  to the output level which maximizes system-wide profits  $E(\beta)(S(q) - \theta_k q)$ . This shows that ex post efficiency is at odds with the purchaser's ex ante desires to maximize his own profits, which is commonly recognized as the lack of

renegotiation-proofness in adverse selection problems (see, for example, Laffont and Tirole (1993) and Laffont and Martimort (2002)). The expectation of renegotiation would, however, destroy the suppliers' incentives to bid according to the initial contract. Therefore the purchaser must be committed to abide by the original terms of the contract. Alternatively, the contracted output could maximize system-wide profits. The supply contract

$$E(\beta)(S'(q) - \theta_k = 0 \tag{12}$$

would be renegotiation-proof. The adaption of (12) would lead to renegotiation-proof product volume, but would be unprofitable to the purchaser, because the payment  $P(q)$  and so informational rents paid to the winning supplier would become excessively large.

The condition (11) also shows that  $q^*(x_k)$  decreases in  $x_k$  and, on the ground of Assumption 2,  $q^*(x_k) > 0$  ( $x_k \in [\underline{\theta}, \theta^o]$ ). The supplier  $k$  will thus always find it worthwhile to participate when  $x_k \leq \theta^o$ . Also taking into account the fact that with probability  $(1 - F(\theta^o))^n$  no producer bids, as a consequence of which the public provider is itself the producer, we obtain for the purchaser's overall expected payoff in the maximum the expression

$$E_{\theta}^{\theta^o}(V1(\theta_k, n)) := (1 - F(\theta^o))^n E_{\beta}(\widehat{WG}(\theta_g)) + E_{\theta}^{\theta^o}(\widehat{WP}(\theta_k)), \tag{13}$$

where  $E_{\beta}(\widehat{WG}(\theta_g))$  is from (2) and  $E_{\theta}^{\theta^o}(\widehat{WP}(\theta_k))$  denotes the expression (10) in which  $q(x_k) = q^*(x_k)$ .

### 3.3 Comparison

We next explore how  $\beta$ 's randomness and the number of bidders affect  $\theta^o$ 's value and the comparative advantages of private and public production. It is also interesting to see whether the public provider sets  $\theta_g > \theta^o$ , which would raise the requirement for the private producer's minimum cost efficiency above the public producer's own cost efficiency. It is also evident that public production brings for the public purchaser a larger outcome than private production if  $\theta_g = \theta_k$  (= the winning private supplier's efficiency).

**Proposition 1** *Given  $\underline{\beta} + \bar{\beta} = A$ , and increase in the spread  $\bar{\beta} - \underline{\beta}$  decreases  $\widehat{WP}(\theta^o)$  in (13) in relation to  $E_\beta(\widehat{WG}(\theta_g))$ . This makes the public provider set  $\theta^o$  below  $\theta_g$  even if  $\bar{\beta} = \underline{\beta}$ , and an increase in  $\bar{\beta} - \underline{\beta}$  forces  $\theta^o$  further down in relation to  $\theta_g$ .*

**Proof.** The proof is divided into 2 parts.

**Part A)** Differentiating (13) with respect to  $\theta^o$ , the  $E_\beta^{\theta^o}(V1(x_k, n))$  is maximized for some value of  $\theta^o$ , satisfying the condition

$$E_\beta(\widehat{WG}(\theta_g)) - \widehat{WP}(\theta^o) = 0. \quad (14)$$

We first show that an increase in  $\bar{\beta} - \underline{\beta}$ , given  $\bar{\beta} + \underline{\beta}$ , increases  $E_\beta(\widehat{WG}(\theta_g))$  but leaves  $\widehat{WP}(\theta^o)$  unchanged. Let us write the integral  $\int_{\underline{\beta}}^{\bar{\beta}}$  in the form  $\int_{A-\bar{\beta}}^{\bar{\beta}}$  and consider the changes in  $E_\beta(\widehat{WG}(\theta_g))$ . We obtain  $\frac{\partial \widehat{WG}(\theta_g)}{\partial \beta} = S(q^{**}) > 0$ ,  $\frac{\partial^2 \widehat{WG}(\theta_g)}{\partial \beta^2} = -\frac{(S'(q^{**}))^2}{\beta S''(q^{**})} > 0$  which shows that  $\widehat{WG}(\theta_g)$  is increasing and strictly convex in  $\beta$ . Owing to this,  $\frac{1}{2}(\widehat{WG}(\theta_g)|_{\beta=\bar{\beta}} + \widehat{WG}(\theta_g)|_{\beta=\underline{\beta}}) > E_\beta(\widehat{WG}(\theta_g)) = \frac{1}{\bar{\beta}-\underline{\beta}} \int_{\underline{\beta}}^{\bar{\beta}} \widehat{WG}(\theta_g) d\beta$ , and so  $\frac{\partial E_\beta(\widehat{WG}(\theta_g))}{\partial \beta} = \frac{1}{\bar{\beta}-\underline{\beta}}(\widehat{WG}(\theta_g)|_{\beta=\bar{\beta}} + \widehat{WG}(\theta_g)|_{\beta=\underline{\beta}}) - \frac{2}{(\bar{\beta}-\underline{\beta})^2} \int_{\underline{\beta}}^{\bar{\beta}} \widehat{WG}(\theta_g) d\beta > 0$ , given  $\underline{\beta} + \bar{\beta} = A$ . This

shows that an increase in  $\bar{\beta} - \underline{\beta}$  increases  $E_{\beta}(\widehat{WG}(\theta_g))$ . On the other hand,  $\frac{\partial \widehat{WP}(\theta^o)}{\partial \beta} = \frac{\partial E_{\theta}(\widehat{WP}(\theta^o))}{\partial \beta} = 0$ . The widening of  $\underline{\beta} - \bar{\beta}$  thus increases  $E_{\beta}(\widehat{WG}(\theta_g))$  in relation to  $\widehat{WP}(\theta^o)$ .

**Part B).** A closer look at  $\widehat{WP}(\theta^o) = E(\beta)S(q^*(\theta^o)) - (\theta^o + \frac{F(\theta^o)}{F'(\theta^o)})q^*(\theta^o)$  reveals that

$$\widehat{WP}(\theta^o) = E_{\beta}(\widehat{WG}(\theta^o)) - \frac{F(\theta^o)}{F'(\theta^o)}q^*(\theta^o) \quad (15)$$

when  $\underline{\beta} = \bar{\beta}$ . From equation (15) it follows that  $\widehat{WP}(\theta^o) < E_{\beta}(\widehat{WG}(\theta^o))$  already when  $\underline{\beta} = \bar{\beta}$ . This result and the fact that by Assumption 3  $\widehat{WP}(\theta^o)$  decreases in  $\theta^o$  implies that  $\theta^o < \theta_g$  when  $\bar{\beta} = \underline{\beta}$ . On the other hand, the finding in Part A) showed that the difference  $E_{\beta}(\widehat{WG}(\theta_g)) - \widehat{WP}(\theta^o)$  only increases in the spread  $\bar{\beta} - \underline{\beta}$ . From this it follows that the validity of (14) requires that  $\theta^o$  must decrease when  $\bar{\beta} - \underline{\beta}$  increases. ■.

The above finding, according to which it pays to set  $\theta^o < \theta_g$  even if  $\bar{\beta} = \underline{\beta}$ , is similar to the previous result of Riley and Samuelson (1981), who showed that the reserve price, being the lowest price at which an item will be sold, is well above the auctioneer's own valuation. This result emerged because the informational rent in the form  $\frac{F(\theta^o)}{F'(\theta^o)}$  lifts the virtual cost paid to the winning supplier above this producer's actual cost  $\theta_k$ , and so the value of  $\theta^o$  where the public provider's marginal benefits of outsourcing the production to type  $k$  equal the marginal benefits of producing itself at the efficiency  $\theta_g$  is below  $\theta_g$ . The auction mechanism thus requires that the private producer is more cost efficient than the public body itself even in the absence of uncertainty related to  $\beta$ . The increase of the variation of unverifiable  $\beta$  only forces  $\theta^o$  lower in relation to  $\theta_g$ .

It is noteworthy that, according to (14), the optimal  $\theta^o$  is also independent of the number of bidders as in Riley and Samuelson (1981). This does not, however, mean that the expected outcome as concerns the decision to provide in-house or out-house would be

independent of the number of bidders.

**Proposition 2** *In the optimum in which (14) is satisfied,  $E_{\theta}^{\theta^o}(V1(\theta_k, n))$  increases in  $n$ .*

**Proof.** *We show that  $E_{\theta}^{\theta^o}(V1(\theta_k, n + 1)) - E_{\theta}^{\theta^o}(V1(\theta_k, n)) > 0$ . This inequality can be written as*

$$[(1-F(\theta^o))^{n+1} - (1-F(\theta^o))^n]E_{\beta}(\widehat{WG}(\theta_g)) + \int_{\underline{\theta}}^{\theta^o} [f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)]\widehat{WP}(x_k)dx_k > 0, \quad (16)$$

where  $f_{(1)}^n(x_k) = nF(x_k)(1 - F(x_k))^{n-1}$ . It can be shown that

$$\int_{\underline{\theta}}^{\theta^o} [f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)]\widehat{WP}(x_k)dx_k > \int_{\underline{\theta}}^{\theta^o} [f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)]\widehat{WP}(\theta^o)dx_k, \quad (17)$$

because  $\widehat{WP}(x_k)$  and  $f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)$  decrease in  $x_k$ ,  $\int_{\underline{\theta}}^{\theta^o} [f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)]dx_k > 0$  and  $\widehat{WP}(x_k)$  is non-negative by Assumption 2. Noticing that  $\int_{\underline{\theta}}^{\theta^o} [f_{(1)}^{n+1}(x_k) - f_{(1)}^n(x_k)]\widehat{WP}(\theta^o)dx_k = [(1 - F(\theta^o))^{n+1} - (1 - F(\theta^o))^n]\widehat{WP}(\theta^o)$  and taking into account (17) and the result (14), we can show that the left-hand side of (16) is above  $F(\theta^o)(1 - F(\theta^o))^n[\widehat{WP}(\theta^o) - E_{\beta}(\widehat{WG}(\theta_g))]$  which is zero. According to this,  $E_{\theta}^{\theta^o}(V1(\theta_k, n + 1)) - E_{\theta}^{\theta^o}(V1(\theta_k, n)) > 0$ . ■

The purchaser's pay-off in out-house provision increases in  $n$  for two reasons. First, the expected cost efficiency of the winning producer increases in  $n$  and, secondly, competition becomes more intense when  $n$  increases, as a consequence of which information rents paid to the out-house provider decrease. This implies that an increase in  $n$  also increases the probability of outsourcing. Indeed, given  $\theta^o$ , the the probability that  $\min\{\theta_1, \dots, \theta_n\}$  is below  $\theta^o$  is  $\int_{\underline{\theta}}^{\theta^o} [f_{(1)}^n(x_k)dx_k = 1 - (1 - F(\theta^o))^n$ , which proves that an increase in  $n$  also increases the probability that the winning bid will be accepted.

## 4 Monitoring in private provision

### 4.1 Monitoring investments and auctioning the opportunity to bargain

We now focus on the situation in which the purchaser invests in monitoring. The monitoring investment  $h$  is taken after the auction, but before the effectiveness parameter  $\beta$  materialises. The right-to-produce and the opportunity to bargain over the additional benefit are auctioned simultaneously.

Bargaining will occur only after the winning supplier is selected. But an opportunity to obtain additional benefit through bargaining will, in any case, reflect in those terms on which the right-to-produce is auctioned. The investment in  $h$  is relation-specific and sunk at the moment of the actual bargain. From this it follows that the investment costs associated with  $h$  are solely carried by the purchaser alone and that the right to bargain is to be sold in conjunction with the regular right-to-produce auction. An option to adjust the pre-specified output target via bargaining is sold in an auction at the fixed price, denoted by  $c(\theta_k)$ , because it is not possible to fix the target level for  $q$  before  $\beta$  materialises. The auctioned values of both  $c(x_k)$  and the quantity-payment schedule  $P(q(x_k))$  depend on the output schedule  $q(x_k)$  and on the planned  $h(x_k)$ . Because the ex ante efficient  $q(x_k)$  and  $h(x_k)$  depart from the respective ex post efficient levels, the purchaser must be committed to also implementing both the ex ante efficient  $q(x_k)$  and the ex ante efficient  $h(x_k)$  to guarantee the success of the auction. The winner in this kind of extended auction is again the producer whose cost parameter  $\theta_k$  is the lowest.

In the extended model, which includes monitoring investments, the order of events is the following: Nature selects  $\theta_k$  ( $k = 1, \dots, n$ ) and  $\theta_g$ . After this a purchaser defines

the quantity-payment schedule  $P(q(x_k))$ , the schedule  $h(x_k)$  for monitoring investments and fixed remuneration  $c(x_k)$  and arranges a sealed high-bid auction in which the right to produce is sold to the firm  $k$  which bids the highest  $q(x_k)$  if  $q(x_k) > q(\theta^o)$ . After the auction the purchaser then invests an amount  $h(x_k)$  in monitoring apparatus. If this investment corresponds to the contracted amount, the private producer will pay the purchaser the amount  $c(x_k)$  for the right to bargain about the possible additional benefits. After this the production phase starts and the effectiveness parameter  $\beta$  materialises. Either the winning firm or the public provider is the producer. In the case in which the auction has selected a private producer, he/she is paid the amount  $P(q(x_k))$ , if the quantity target specified for  $q(x_k)$  is met. Otherwise, the producer is punished. If the purchaser also succeeds in observing  $\beta$ , the parties will adjust the output at the optimal level and the winning firm will obtain its share of an additional gain in addition to  $P(q(x_k))$ .

That part of the system's expected payoff which accrues from the right-to-produce contract (in the absence of bargaining) is still given in (3) and denoted by  $E_{\theta}^{\theta^o}(SWP1(\theta_k))$ . Let  $q_b$  denote the output which is adjusted according to materialized  $\beta$ . After the auction and after the value of  $\beta$  has been resolved, the system-wide payoff in the private production,  $SWP2(\theta_k) := \beta S(q_b) - \theta_k q_b$ , is mutually maximized with respect to  $q_b$ , which yields the condition

$$\beta S'(q_b) - \theta_k = 0 \tag{18}$$

for  $q_b^o(\theta_k) := \arg \max_{q_b} SWP2(\theta_k)$ .

**Assumption 4.**  $q(\theta_k)$  defined by (18), being also a function of  $\beta$ , is convex in  $\beta$ .

This assumption is valid, if  $S'''(q)S'(q) - (S''(q))^2 \geq 0$ . It is then valid for many typical

functional forms of  $S(q)$ . Assumption 4 guarantees that the second order conditions for incentive compatibility are in force.

Because  $\beta$  is not verifiable, the exact value of  $q_b^o(\theta_k)$  - which is a function of  $\beta$  and thus materializes after  $\beta$  resolves - cannot be contracted upon and made contingent on  $\beta$ . The expected value of  $SWP2(\theta_k)$  before the auction, and so also before  $\beta$  materializes, is then

$$E_{\theta,\beta}^{\theta^o}(SWP2(\theta_k)) = \int_{\underline{\theta}}^{\theta^o} \int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - x_k q_b^o(x_k)] G'(\beta) f_{(1)}(x_k) d\beta dx_k.$$

The parties - the purchaser and the private producer - will bargain over the split of an additional benefit  $SWP2(\theta_k) - SWP1(\theta_k)$  with probability  $p(h)$  when  $SWP1(\theta_k)$  is defined in (3). The purchaser obtains a share  $\lambda$ , ( $0 \leq \lambda \leq 1$ ) of this benefit. Making bids on the supply contract, potential private suppliers are also aware of the future level of  $h(x_k)$ . The expected overall system-wide profits from private production before the auction, denoted by  $E_{\theta,\beta}^{\theta^o}(SWP3(\theta_k))$ , have the equation

$$E_{\theta}^{\theta^o}(SWP3(\theta_k)) = (1 - p(h(x_k)))E_{\theta}^{\theta^o}(SWP1(\theta_k)) + p(h(x_k))E_{\theta,\beta}^{\theta^o}(SWP2(\theta_k)) - h(x_k). \quad (19)$$

Let us then analyse more closely the implications of an auction in which the right-to-produce and the opportunity to bargain over the additional benefit are auctioned simultaneously.

The supplier  $k$ 's total payoff  $\Pi_b^k(\theta_k, x_k)$  is then

$$\begin{aligned} \Pi_b^k(\theta_k, x_k) = & \{P(q(x_k)) - \theta_k q(x_k) + p(h(x_k))[(1 - \lambda)(E_{\beta}(SWP2(\theta_k, x_k)) - SWP1(\theta_k, x_k))] \\ & - c(x_k)\}(1 - F(x_k))^{n-1} \end{aligned} \quad (20)$$

where  $E_{\beta}(SWP2(\theta_k, x_k)) = \int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - \theta_k q_b^o(x_k)] G'(\beta) d\beta$  and  $SWP1(\theta_k, x_k) = E(\beta)S(q(x_k)) -$

$\theta_k q(x_k)$ . The supplier's income in (20) consists of three parts. The first is  $P(q(x_k)) - \theta_k q(x_k)$ , the income from the right-to-produce contract. The second is  $p(h)(1-\lambda)(E_\beta(SWP2(\theta_k, x_k)) - SWP1(\theta_k, x_k))$ , the share of the additional income which accrues, if output is adjusted according to actual  $\beta$ . The winning supplier must also pay  $c(x_k)$  for the right to bargain despite the success in monitoring  $\beta$ .

The auction induces supplying firms to reveal their type and maximise  $\Pi_b^k(\theta_k, x_k)$  with respect to  $x_k$  so that  $\frac{\partial \Pi_b^k(\theta_k, x_k)}{\partial x_k} = 0$ . Applying the envelope theorem, we obtain from (20)

$$\left. \frac{\partial \Pi_b^k(\theta_k, x_k)}{\partial \theta_k} \right|_{x_k=\theta_k} = -[q(x_k) + p(h(x_k))(1-\lambda)(E_\beta(q_b(x_k)) - q(x_k))](1-F(x_k))^{n-1}, \quad (21)$$

where  $E_\beta(q_b(x_k)) := \int_{\underline{\beta}}^{\bar{\beta}} q_b^o(x_k) G'(\beta) d\beta$ . The second-order condition for the incentive compatibility is given in the Appendix. Because  $\frac{\partial \Pi_b^k}{\partial \theta_k} = -[1 - p(h(x_k))(1-\lambda)]$  preserves its sign, the sufficient condition for implementation is also valid.

Define  $\Pi_b^k(\theta_k) = \Pi_b^k(\theta_k, \theta_k)$ . Setting  $\Pi_b^k(\theta^o) = 0$ , one obtains from (21)

$$\Pi_b^k(\theta_k) = \int_{\theta_k}^{\theta^o} [q(x_k) + p(h(x_k))(1-\lambda)(E(q_b(x_k)) - q(x_k))](1-F(x_k))^{n-1} dx_k. \quad (22)$$

The supplier  $k$ 's expected profits derived from (22) are

$$E_\theta^{\theta^o}(\Pi_b^k(\theta_k)) = \int_{\underline{\theta}}^{\theta^o} [q(x_k) + p(h(x_k))(1-\lambda)(E(q_b(x_k)) - q(x_k))](1-F(x_k))^{n-1} F(x_k) dx_k. \quad (23)$$

The sum of all  $n$  producer's expected profits is  $nE_\theta^{\theta^o}(\Pi_b^k(\theta_k))$ .

The purchaser's expected profits, denoted,  $E_\theta^{\theta^o}(V2(\theta_k))$ , are then

$$E_\theta^{\theta^o}(V2(\theta_k, n)) = (1-F(\theta^o))^n E_\beta(\widehat{WG}(\theta_g)) + \int_{\underline{\theta}}^{\theta^o} WPU(x_k) f_{(1)}(x_k) dx_k, \quad (24)$$

where  $\int_{\underline{\theta}}^{\theta^o} WPU(x_k) f_{(1)}(x_k) dx_k := E_{\theta, \beta}^{\theta^o}(SWP3(\theta_k)) - nE_\theta^{\theta^o}(\Pi_b^k(\theta_k))$  when  $E_{\theta, \beta}^{\theta^o}(SWP3(\theta_k))$

is defined in (19) and  $E_{\theta}^{\theta^o}(\Pi_b^k(\theta_k))$  in (23).  $WPU(x_k)$  in (24) then has the equation

$$\begin{aligned} WPU(x_k) &= (1 - p(h(x_k)))[E(\beta)S(q(x_k)) - x_k q(x_k)] \\ &\quad + p(h(x_k))\left[\int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - x_k q_b^o(x_k)] G'(\beta) d\beta\right. \\ &\quad \left. - [(1 - p(h(x_k)))(1 - \lambda))q(x_k) + p(h)(1 - \lambda)(E(q_b^o(x_k)))] \frac{F(x_k)}{F'(x_k)} - h(x_k)\right] \end{aligned} \quad (25)$$

The assumptions of the model, for the present, guarantee that  $E_{\theta}^{\theta^o}(V2(\theta_k, n))$  is concave in  $q(x_k)$  given  $h$ , and, respectively, that it is concave in  $h(x_k)$  given  $q(x_k)$ . However, next we assume that  $E_{\theta}^{\theta^o}(V2(\theta_k, n))$  is also jointly concave in  $q(x_k)$  and  $h(x_k)$ .

**Assumption 5.**  $E_{\theta}^{\theta^o}(V2(\theta_k, n))$  in (24) is jointly concave in  $q(x_k)$  and  $h(x_k)$ .

By Assumption 5, we can focus on the first-order conditions while considering the optimal conduct in terms of  $q(x_k)$  and  $h(x_k)$ .

Differentiating  $WPU(x_k)$  from (25) with respect to  $q(x_k)$  and  $h(x_k)$  we obtain the first-order condition

$$E(\beta)S'(q(x_k)) - x_k - \frac{F(x_k)}{F'(x_k)} \left( \frac{1 - p(h(x_k))(1 - \lambda)}{(1 - p(h(x_k)))} \right) = 0, \quad (26)$$

which defines  $q^+(x_k) \equiv \arg \max_q E_{\theta}^{\theta^o}(V2(\theta_k, n))$  and the condition

$$\begin{aligned} &p'(h(x_k)) \left\{ \int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - x_k q_b^o(x_k)] G'(\beta) d\beta - [E(\beta)S(q^+(x_k)) - x_k q^+(x_k)] \right\} \\ &- (1 - \lambda) \frac{F(x_k)}{F'(x_k)} \left( \int_{\underline{\beta}}^{\bar{\beta}} q_b^o(x_k) G'(\beta) d\beta - q^+(x_k) \right) \} - 1 \\ &= 0, \end{aligned} \quad (27)$$

which defines  $h(x_k)^+ \equiv \arg \max_h E_{\theta}^{\theta^o}(V2(\theta_k, n))$ .

After the winning supplier has been chosen, it is tempting for the winning supplier and the purchaser to abstract from incentive compatibility and readjust  $P(q(x_k))$  and  $h(x_k)$ .

The supply contract  $q^+(x_k), h^+(x_k)$  is thus not renegotiation-proof and the purchaser's commitment to stick to the initial payment schedules and monitoring investments are required to deter renegotiations. We next consider solutions (26) and (27) and assume that the purchaser has enough commitment capacity to keep to the initial contract.

## 4.2 Output decision

We will show that the opportunity to get some extra benefit through monitoring investments increases, in any case, the output. From (26) we obtain the following result:

**Proposition 3** *When  $h > 0$  and  $\lambda > 0$  an opportunity to obtain an extra benefit through bargaining increases output, given  $x_k$ , even if  $\beta$  is ex post not mutually observed.*

**Proof.** Comparing  $q^+(x_k)$ , defined by (26) with  $q^*(x_k)$  defined by (11), proves that  $q^+(x_k) > q^*(x_k)$ , because  $1 - \lambda + \frac{\lambda}{(1-p(h))} > 1$  in (26) and  $S'(q(x_k))$  decreases in  $q(x_k)$ . ■

The result obtained describes the fact that already a positive probability to achieve some extra benefit thorough bargaining raises the marginal benefits of production and therefore also the level of production. The opportunity to monitor  $\beta$  then corrects the downward bias of  $q(x_k)$  which is created by the pre-fixing of  $q(x_k)$  and concavity of  $S(q(x_k))$ .

If  $\beta$  is actually observed,  $E_\beta(q_b^o(x_k))$  will arise above  $q^+(x_k)$ .

**Proposition 4** *When  $h > 0$  and  $\lambda > 0$  and when the monitoring leads to the exposure of  $\beta$ , the expected output level rises above  $q^+(x_k)$ , given  $x_k$ .*

**Proof.** The proof is given in two stages.

First stage: compare the condition (26) with the condition

$$E(\beta)S'(q(x_k)) - x_k = 0, \quad (28)$$

which defines  $q^a(x_k)$ . From this one sees directly that  $q^+(x_k)$  is below  $q^a(x_k)$ .

Second stage: Let  $q^a(E(\beta))$  denote  $q^a(x_k)$ , given  $x_k$  and  $E(\beta)$  and  $q_b^o(\beta)$  denote  $q_b^o(x_k)$ , given  $x_k$  and  $\beta$ . We obtain from (28) and (18) the equation  $E(\beta)S'(q^a(E(\beta))) = \beta S'(q_b^o(\beta))$ . From this equation it follows that

$$E(\beta)S'(q^a(E(\beta))) = \int_{\underline{\beta}}^{\bar{\beta}} \beta S'(q_b^o(\beta))G'(\beta)d\beta. \quad (29)$$

On the other hand, from (18) is also obtained the following:

$$E(\beta)S'(q_b^o(E(\beta))) = \int_{\underline{\beta}}^{\bar{\beta}} \beta S'(q_b^o(\beta))G'(\beta)d\beta. \quad (30)$$

By Assumption 4,

$$\int_{\underline{\beta}}^{\bar{\beta}} \beta q_b^o(\beta)G'(\beta)d\beta \geq q_b^o(E(\beta)).$$

Using the above inequality and equation (30) we obtain

$$E(\beta)S'(E_\beta(q_b^o(\beta))) \leq \int_{\underline{\beta}}^{\bar{\beta}} \beta S'(q_b^o(\beta))G'(\beta)d\beta.$$

This inequality together with (29) implies that

$$E(\beta)S'(E_\beta(q_b^o(\beta))) \leq E(\beta)S'(q^a(E(\beta)))$$

from which it follows that

$$E_\beta(q_b^o(\beta)) \geq q^a(E(\beta)).$$

Because  $q^a(x_k) > q^+(x_k)$ , then also  $E_\beta(q_b^o(x_k)) > q^+(x_k)$ . ■

The probable success in monitoring removes the allocative inefficiency associated with the inability to respond to actual  $\beta$ , which raises the output in the quantity payment schedule. Owing to some additional informational rents which increase marginal costs by  $(1 - \lambda) \frac{F(x_k)}{F'(x_k)} (E_\beta(q_b^o(x_k)) - q^+(x_k))$  in (27),  $q^+(x_k)$  still remains below  $E_\beta(q_b^o(x_k))$ .

### 4.3 Monitoring decision

The assumptions of the model do not guarantee that the inner-point solution of the condition (27) defines a maximum. If the marginal benefits of monitoring investment, which accrue when  $q$  is adjusted according to the mutually observed  $\beta$ , are small in relation to the respective marginal costs whose size is  $-1$ , the left-hand side of (27) can be negative with all  $\theta_k < \theta^o$ . The investments in monitoring would then be zero. Next we consider a situation in which it pays to invest in monitoring.

**Proposition 5** . *The monitoring investments shift  $\theta^o$  closer to  $\bar{\theta}$  if  $h^+(\theta^o) > 0$ .*

**Proof.** Maximizing  $E_\theta^{\theta^o}(V2(x_k, n))$  in (24) with respect to  $\theta^o$ , gives the first-order condition

$$E_\beta(\widehat{WG}(\theta_g)) - WPU(\theta^o) = 0 \quad (31)$$

for  $\theta^o$ . In this condition  $E_\beta(\widehat{WG}(\theta_g))$  is independent of  $\theta^o$  and by Assumption 3  $WPU(\theta^o)$  decreases in  $\theta^o$ . On the other hand, when  $h^+(\theta^o) > 0$ ,  $WPU(\theta^o)$  in (24) increases in  $h(\theta^o)$  when  $0 < h(\theta^o) < h^+(\theta^o)$ . From this it follows that an increase in  $h(\theta^o)$ , while  $h(\theta^o)$  is still below  $h^+(\theta^o)$ , must be balanced by increasing  $\theta^o$  to make (31) valid ■.

The monitoring opportunity then easily enlarges the scope for private provision. In the situation in which  $h^+(x_k) > 0$ , it is also interesting to compare  $h^+(x_k)$  defined by

(27) with a socially optimal  $h(x_k)$ , denoted by  $h^s(x_k)$ , which maximizes the social welfare  $SWP3(\theta_k) = (1 - p(h(x_k)))[E(\beta)S(q^+(x_k)) - \theta_k q^+(x_k)] + p(h(x_k)) \int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - \theta_k q_b^o(x_k)] G'(\beta) d\beta - h(x_k)$ . The condition

$$p'(h(x_k)) \int_{\underline{\beta}}^{\bar{\beta}} [\beta S(q_b^o(x_k)) - \theta_k q_b^o(x_k)] G'(\beta) d\beta - [E(\beta)S(q^+(x_k)) - \theta_k q^+(x_k)] - 1 = 0 \quad (32)$$

then defines  $h^s(x_k) \equiv \arg \max_h SWP3(\theta_k)$ . We obtain the following proposition concerning the level of monitoring investments:

**Proposition 6** *Part A) When  $h^s(x_k) > 0$  and  $\lambda < 1$ , the purchaser sets  $h^+(x_k)$  below  $h^s(x_k)$ . Part B) When  $h^+(x_k) > 0$ , the distortion in  $h^+(x_k)$  decreases in  $\lambda$ .*

**Proof.** Part A) Because  $\int_{\underline{\beta}}^{\bar{\beta}} q_b^o(x_k) G'(\beta) d\beta - q^+(x_k) > 0$  in (27) and because  $p'(h(x_k))$  decreases in  $h(x_k)$ ,  $h^+(x_k)$  defined by (27) must be smaller than  $h^s(x_k)$ .

Part B) It is straightforward that an  $(1 - \lambda) \frac{F(x_k)}{F'(x_k)} (\int_{\underline{\beta}}^{\bar{\beta}} q_b^o(x_k) G'(\beta) d\beta - q^+(x_k))$  in (27) decreases in  $\lambda$  which makes  $h^+(x_k)$  become closer to  $h^s(x_k)$ . ■

The result obtained shows that the purchaser's bargaining power can even promote social welfare because the "hold-up" weakens the purchaser's (the government's) own investments. The previous results (see, for example, Hart et al., 1997) focused on the suppliers' incentives to invest in innovations. Then the purchaser's bargaining power improves the social welfare insofar as the positive cost savings effect dominates the negative quality deteriorating effect.

#### 4.4 Solving $P(q(\theta_k))$ and $c(\theta_k)$

Expressions for  $P(q(\theta_k))$  and  $c(\theta_k)$  in (20) can be solved in terms of  $q^+(\theta_k)$  and  $q_b^o(\theta_k)$  by first replacing  $q(x_k)$  by  $q^+(\theta_k)$  and  $q_b(x_k)$  by  $q_b^o(\theta_k)$  in (20). Writing expression (20) then

in the form  $\Pi_b^k(\theta_k, \theta_k)$  and equating the right hand-side of the equation obtained with the right-hand side of (22), one can solve for expressions for  $P(q^+(\theta_k))$  and  $c(\theta_k)$ . For the quantity-payment schedule  $P(q^+(\theta_k))$  one again obtains the equation (8). For the price of the bargaining opportunity we obtain the equation

$$c(\theta_k) = p(h(\theta_k))[(1 - \lambda)(SWP2(\theta_k) - SWP1(\theta_k)) - \frac{\int_{\theta_k}^{\theta^o} (1 - \lambda)(E(q_b^o(x_k)) - q^+(x_k))(1 - F(x_k))^{n-1} dx_k}{(1 - F(\theta_k))^{n-1}}]. \quad (33)$$

The above equation shows that by auctioning the bargaining opportunity, the purchaser is able to obtain the expectation for the supplier's share of the ex post bargain minus the informational rent, which is the second term on the right-hand side of (33).

## 5 Conclusions

This study has shown that the choice between private or public production leads to a trade-off between cost efficiency - which improves when competition intensifies - and, on the other hand, effectiveness - which is the measure of the extent to which the system responds to unforeseen individual needs. According to the central assumption of this analysis, only that unit which produces learns the individuals needs of customers. The informational asymmetry which has arisen and which concerns effectiveness is typical of the production of welfare services. In the outsourcing of such public goods as, for example, administrative tasks, the informational incompleteness is different. The public provider may constantly be better informed about productional requirements than the possible out-house producer. The same may concern private firms in the situations in which they outsource core tasks. In the latter situations the implications of outsourcing

may differ from the results obtained in this study. But in all situations the unverifiable nature of effectiveness may cause similar contractual problems: it is very hard to induce the out-house producer to allocate his activities in an effective way.

On the whole, by focusing on the unverifiable and unforeseen nature of individual needs which service production should satisfy, we introduce new insight into the topic under consideration. Our study can also explain the recent development in some western countries. The tendency toward contracting out, in the field of, for example, welfare services, has been rather slow in those industrialised countries in which the state and municipalities still produce a huge part of these services themselves. The tendency toward private production is slowed down by the cautious attitude of political decision-makers and authorities. Maybe the experiences of private production do not encourage them to take big steps toward the privatisation of these services. On the other hand, in the field of technical services - which used to be quite widely produced by the state and municipalities - the pace toward private production has been rather fast.

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## 6 Appendix

The second-order condition related to (21) requires that the expression

$$\begin{aligned} & -(1 - p(h)(1 - \lambda)) \frac{\partial(q^+(x_k)(1 - F(x_k))^{n-1})}{\partial x_k} - p(h)(1 - \lambda) \frac{\partial(E_\beta q^o(x_k)(1 - F(x_k))^{n-1})}{\partial x_k} \\ & -(1 - \lambda)(E_\beta(q_b^o(x_k)) - q^+(x_k)) \frac{\partial(p(h(x_k)(1 - F(x_k))^{n-1})}{\partial x_k}. \end{aligned}$$

for  $\frac{\partial^2 \Pi_k^k}{\partial x_k \partial \theta_k}$  is non-negative. Clearly,  $q^+(x_k)$  defined by (26),  $h(x_k)$  defined by (27) and  $q^o(x_k)$  defined by (18) decrease in  $x_k$ , and so  $(1 - F(x_k))^{n-1}$ . This, and the fact that  $E_\beta(q_b^o(x_k)) > q^+(x_k)$  (see the proof for proposition 4) guarantees that  $\frac{\partial^2 \Pi_k^k}{\partial x_k \partial \theta_k} \geq 0$ .