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TIIVISTELMÄ

Tutkimuksessa analysoidaan Suomen tehdasteollisuuden rakennemuutosta hyödyntämällä tuotantofunktioiden aggregoinnin teoriaa ja teollisuustoimipaikkaneeliaineistoa, joka kattaa ajanjakson 1980–2005. Kahdentoista teollisuustoimialan sisäistä rakennemuutosta tarkastellaan sen perusteella, ovatko toimialoitaiset aggregaattituotantofunktiot pysyneet tarkasteluajavälillä muuttumattomina. Aggregaattituotantofunktioita ei tarvitse estimoida, koska tuotantofunktioiden aggregoinnin teorian perusteella funktioiden muuttumattomuutta voidaan analysoida tarkastelemalla kunkin toimialan arvonlisäyksen jakaantumista kuvaavansa kapasiteetin tiheysfunktion muodon pysyvyyttä. Vaikka aggregaattituotantofunktion muoto muuttuu tutkimustulosten perusteella jokaisella toimialalla, muutosten ajoittuminen ja voimakkuus vaihtelevat huomattavasti. Tutkimus esimerkiksi vahvistaa jo aiemmin saadun tuloksen, jonka mukaan 1980-luvun jälkipuolisko oli paperiteollisuudessa rakennemuutoksen aikaa. Elintarviketeollisuus ja tietoliikennevälineitä valmistava teollisuus ovat esimerkkejä toimialoista, joissa rakennemuutos oli voimakasta erityisesti 1990-luvulla.

Asiasanat: teknologinen muutos, rakennemuutos, aggregoinnin teoria, tuotantofunktio, betaregressio

ABSTRACT

In the paper, structural change in the Finnish manufacturing industries is studied by means of the theory of the aggregation of production functions and longitudinal plant-level data for the period from 1980 until 2005. The nature of structural change in twelve industries is characterised by examination of the invariance of the aggregate production functions over time. Aggregate production functions need not be estimated because, according to the theory of the aggregation of production functions, the invariance can be analysed by the investigation of the stability of the capacity density functions which describe the distribution of value added in the industries. Even though the shapes of aggregate production functions alter over time in most industries, there are differences in timing and in the degree of turbulence across industries. The analysis confirms the result obtained earlier that in some industries, for example in the paper industry, the late 1980s marked the beginning of a period of relatively strong structural change. The food industry and the manufacture of communications equipment are examples of industries in which the 1990s was a period of turbulence.

JEL classification: C43, L16, L60, O33

Key words: technological change, structural change, aggregation theory, production function, beta regression

1. INTRODUCTION

In their survey on the use of longitudinal micro-level data sets (LMDs) in studies on productivity, Bartelsman and Doms (2000) write: “The popularity of this emerging research can be ascribed, in part, to increased availability of micro-level data, to the development of a rich theoretical microeconomic foundation, and to the displeasure of the concept of the aggregate production function.”

A confrontation between the use of LMDs and the use of an aggregate production function seems inevitable due to the great heterogeneity of plants within industries witnessed by LMDs. Certainly the use of the aggregate production function is normally based on the assumption about the representative firm. In this paper, however, our aim is to show that, even in the case in which LMDs are the basic source of information in studies on productivity or in studies on structural change more generally, the use of the concept of the aggregate production function may turn out to be useful. We demonstrate that an analysis with LMDs can make use of the concept of the aggregate production function. Therefore, the concept need not be displeasing but may be expedient.

The most common approach to building a micro-macro link in the productivity analysis which utilises LMDs has been the decomposition of aggregate labour or total factor productivity (TFP) growth into components which make extensive use of the possibilities provided by micro data. Typically, aggregate TFP growth is decomposed into components which are related to within-plant growth, reallocation, and the effects of exit and entry. With such decomposition, the characterisation of aggregate productivity growth can become much richer than one based only on the use of aggregative data and the use of the assumption about the representative firm.

In addition to studies which are based on the above decompositions, micro-macro links have also been built in studies which utilise productivity distributions in examining the degree of dispersion in productivity across plants within industries. For example, a more detailed picture of average productivity growth can be obtained by examining how productivity distributions have evolved (see, for example, Baily et al. 1992).

In this paper, we take a further step in the analysis of distributions of this kind by tying them closely to the theory of production and by utilising them in the characterisation of the smoothness of productivity or technological change. It can be shown that productivity or efficiency distributions provide an invaluable micro-macro link which can be utilised extensively in empirical production analysis which is based on LMDs and standard production theory. The link is provided by the theory of the aggregation of production functions.

Within that theory, macro behaviour in production is related to micro behaviours of heterogeneous firms through productivity or efficiency distributions. Within the theory of the aggregation of production functions, the concept of the aggregate production function is essential. Even though we will not estimate a

single aggregate production function in this paper, we will utilise the concept of the aggregate production function extensively. It will serve as a means of characterisation of technological, or more generally, structural change.

Within the theory of the aggregation of production functions, the invariance of the aggregate production function over time is of great interest. An aggregate production function is invariant over time if its shape does not change. In this paper we utilise the invariance conditions in the description of the nature of technological change. When the above-mentioned decompositions have been used in the characterisation of productivity change, periods of drastic changes like periods which have been regarded as periods of creative destruction have been identified. In this paper, turbulent periods are periods when the invariance conditions do not hold, i.e. the shape of the aggregate production function alters.

The framework which we utilise allows us to give a parametric representation for the nature of technological change for a given period. It therefore complements approaches which utilise non-parametric decompositions of productivity growth. Furthermore, because the framework is directly linked to the standard (neoclassical) theory of production, it may have a closer link to economic theory than the framework which utilises the above-mentioned decompositions. In particular, it attempts to combine statistical distributions of (relative) productivity more closely to the economic theory of production and especially to some theories of structural change.

The aim of this paper is not to make a contribution to the theory of the aggregation of production functions. On the contrary, we make an effort to revive a very important but old and, perhaps for that reason, almost forgotten approach to the theory of the aggregation of production functions. The most important developer of the approach has been Kazuo Sato (1975), who started to develop the theory as early as the late 1960s. Some of the ideas were formulated independently by Leif Johansen (1970). The main source of inspiration for both was the advice given by Robert Solow (1967) to take up and to further develop the idea put forward by Hendrik S. Houthakker (1955-1956). All three regard Houthakker as the originator of the framework, which was also used by David Levhari (1968).¹ In this paper we call the framework Sato's approach and base the presentation of its basic ideas on his book (Sato 1975).

In this paper our main contribution is the application of Sato's approach to the analysis of productivity, or more generally, technological and structural change by the utilisation of LMDs. We use Sato's approach in examining structural change in the Finnish manufacturing industries during the years from 1980 until 2005. The use of Finnish data is beneficial, because one of us (Maliranta) has made a thorough study on a similar subject by using the same data, and by utilising the approach based on the use of one variant of the above-

¹ Jones (2005) is an example of a relatively recent article which utilises the basic insight by Houthakker.

mentioned decompositions (Maliranta 2003). We can therefore compare the results obtained by the use of these two approaches.

Our paper is organised as follows. We first introduce the basic ideas of Sato's approach. After presenting the LMD data we provide a descriptive analysis of the data by utilising some basic concepts introduced by Sato (1975). Thereafter the results of the econometric analysis are presented. We conclude by summarising the results and by pointing to areas in need of further investigation.

2. SATO'S APPROACH AND STRUCTURAL CHANGE

Sato's approach is based on Houthakker (1955-1956), in which Houthakker analysed, within the framework of activity analysis, the aggregation of micro-production functions subject to Leontief technology. Houthakker introduced the idea of the efficiency distribution as the link between micro- and macro-production functions. The shape of the aggregate production function, if it exists, depends on the efficiency distribution. Even though Houthakker's approach is only a special case, it is utilised here in the presentation of the basic ideas of Sato's approach. (See Sato 1975, Ch. 1 and Ch. 2).

Assume a competitive industry which consists of many (a continuum of) firms. They produce homogeneous output (q) by using homogeneous labour (l) and capital (k), which is assumed to be heterogeneous across firms. Each firm utilises only one kind of capital, which, because of the heterogeneity, is not comparable across firms.

Under the assumption of Leontief technology, the production function of the i th firm is given by

$$(1) \quad q_i = \mathbf{a}_i k_i = \mathbf{b}_i l_i.$$

Even though the functional form of the production function is assumed to be common to all firms, the efficiency parameters \mathbf{a}_i and \mathbf{b}_i differ among them.

Besides the product market, the labour market is also assumed to be competitive.² This means that firms take the real wage rate (x) as given. Firms are engaged in operation as long as

$$(2) \quad q_i - x l_i = q_i (1 - x / \mathbf{b}_i) \geq 0 \quad \text{or} \quad \mathbf{b}_i \geq x,$$

² This assumption is not necessary. The basic ideas could also be presented by utilising such wage negotiation models that belong to the class of right-to-manage models. Sato also assumes perfect competition in the product market for the sake of simplicity. That assumption can be relaxed. (See Sato 1975, Ch. 5.)

i.e. as long as their quasi-rents are non-negative.

One of the basic assumptions is that labour input is the only variable input. Capital is assumed to be given in the short run. This also means that the production technique is assumed to be given i.e. the production functions are assumed to be short run or ex post production functions. Efficiency differences reflect heterogeneous capital because techniques are embodied in capital goods. At each point of time a distribution of heterogeneous capital goods exists and it can be expressed in terms of the parameters \mathbf{a} and \mathbf{b} . The distribution, which is denoted by $k(\mathbf{a}, \mathbf{b})$, is assumed to be continuous.

For each value of \mathbf{b} define

$$(3) \quad \mathbf{f}(\mathbf{b}) = \int_{\mathbf{a} \in A} \mathbf{a} k(\mathbf{a}, \mathbf{b}) d\mathbf{a} .$$

For a particular value \mathbf{b} , $\mathbf{f}(\mathbf{b})$ represents the total productive capacity of firms in the industry. Even though capital goods are heterogeneous, $\mathbf{a}k$ represents the amount of homogenous output. Sato calls $\mathbf{f}(\mathbf{b})$ the capacity density function.

The capacity density function is the link between micro and macro functions. Output and labour aggregates can be defined as

$$(4) \quad Q(x) = \int_x^{b_0} \mathbf{f}(\mathbf{b}) d\mathbf{b}$$

and

$$(5) \quad L(x) = \int_x^{b_0} \frac{\mathbf{f}(\mathbf{b})}{\mathbf{b}} d\mathbf{b} ,$$

respectively. In (4) and (5), b_0 is the maximum of the labour efficiency coefficient. In the derivation of the aggregate production function, equations (4) and (5) are used. It can be shown that, if the aggregate production function exists, it must be such that

$$(6) \quad Q = F(J, \mathbf{b}_0 L) \text{ where } J = Q(0) = \int_0^{b_0} \mathbf{f}(\mathbf{b}) d\mathbf{b} .$$

Within Sato's framework the problem of aggregating heterogeneous capital goods is solved by the redefinition of aggregate capital as the total productive capacity of operating firms, or as efficiency-corrected capital stock.

The functional form of the aggregate production function is determined by the capacity density function. Houthakker (1955-56) showed that, if the capacity density function is a Pareto distribution, the aggregate production function is a Cobb-Douglas production function. Sato generalised Houthakker's analysis by, for example, dropping the assumption of Leontief technology. He also analysed the existence of aggregate production functions within a well specified general mathematical framework. The existence problem can be formulated as follows.

Assume that both micro- and macro-production functions are given. The question is: what form must the density function $f(\mathbf{b})$ take in order to make the two consistent? As a mathematical problem, this is a problem of integral equations. Sato (1975) provides a detailed analysis of this existence problem. (See Sato 1975, 20-23.) Levhari (1968) has already given a solution to this problem in a special case, i.e. in the case where micro-production functions are, as in Houthakker (1955-1956), fixed-proportions functions and the aggregate production function is a CES production function with $0 < \mathbf{s} < 1$, where \mathbf{s} is the elasticity of substitution.

For our purpose, the mathematical details of the existence problem are of minor importance. However, it is important to recognize that the correspondence between an aggregate production function and a capacity density function guaranteeing consistency is one to one. This means that, if $f(\mathbf{b})$ fulfils certain regularity conditions, there is an aggregate production function with capital and labour aggregates represented by (6). In the empirical part of our analysis we will utilize this general result even though we may not be interested in the specific form that the aggregate production function takes.

The existence conditions are time-specific. When time passes, the capacity density function $f(\mathbf{b})$ and \mathbf{b}_0 may change. Such changes may alter both J and F . For empirical analysis, it is important to know which kind of changes in $f(\mathbf{b})$ and \mathbf{b}_0 leave F unchanged. Within Sato's framework, this is called the invariance problem. Obviously, only particular shifts in $f(\mathbf{b})$ are allowed if F is to be invariant.

Denoting the shift variable by t and two points of time by t_0 and t_1 it is easy to show that the aggregate production function F remains invariant at these two points of time in the following two cases: the case of scaling-up and the case of stretching-out.

If

$$(7) \quad f(\mathbf{b}, t_1) = a f(\mathbf{b}, t_0), \mathbf{b}_0(t_1) = \mathbf{b}_0(t_0) = \mathbf{b}_0,$$

i.e. if the capacity density function is scaled up proportionally everywhere as depicted by Figures 1a and 1b, the aggregate production function remains invariant.

Figure 1. Scaling-up.

Fig. 1a. Bell-shaped capacity density function

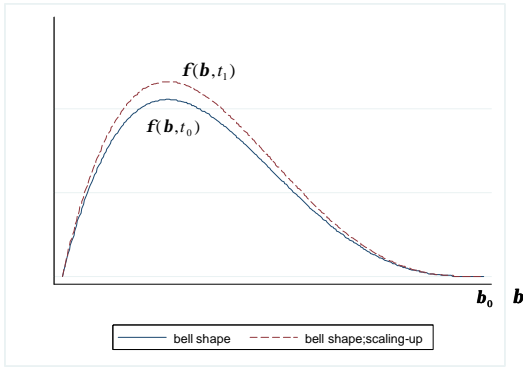
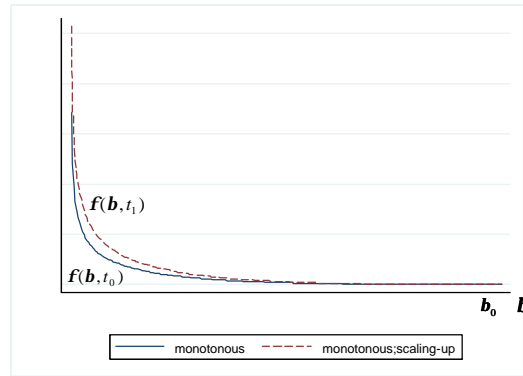


Fig.1b. Monotonous capacity density function



Source: Adapted from Sato (1975).

In this case, the aggregate production functions are

$$(8) \quad Q = F(J(t_i), \mathbf{b}_0 L), \quad i = 1, 2,$$

where $J(t_1) = aJ(t_0)$. Only the capital aggregate has changed. Both increases in k 's and in a 's can cause scaling up. The scaling-up condition is satisfied if proportionate changes are equal in each firm. Capital-augmenting technical change is therefore reflected as the scaling-up of the capacity density function.

Assume that

$$(9) \quad \mathbf{b}_0(t_1) f(\mathbf{b}, t_1) = \mathbf{b}_0(t_0) f\left[\frac{\mathbf{b}_0(t_0)}{\mathbf{b}_0(t_1)} \mathbf{b}, t_0\right].$$

This change, which is depicted by Figures 2a and 2b, represents the stretching-out of the capacity density function.

Figure 2. Stretching-out.

Fig 2a. Bell-shaped capacity density function

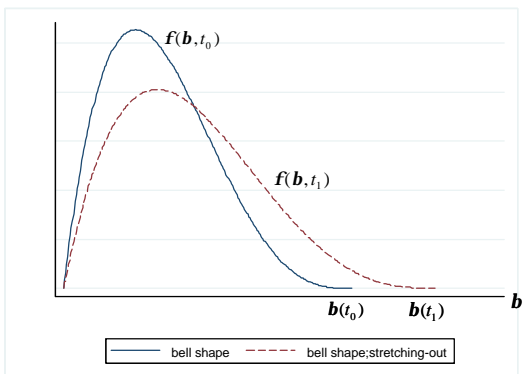
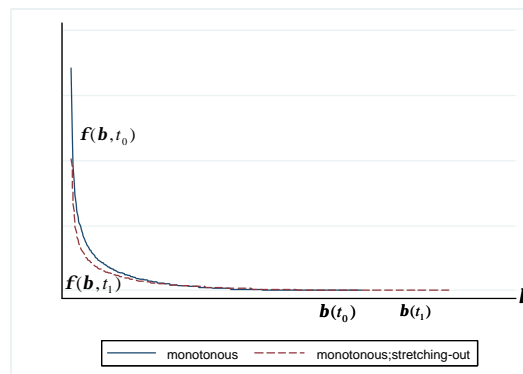


Fig 2b. Monotonous capacity density function



Source: Adapted from Sato (1975).

In the case of stretching-out, labour efficiency coefficients grow at the same pace in each firm. Denoting $\mathbf{b}_0(t_1) = b\mathbf{b}_0(t_0)$, $b > 1$, the aggregate production function corresponding to stretching out can be written as

$$(10) \quad Q = F(J(t_1), \mathbf{b}_0(t_1)L) = F(J(t_0), b\mathbf{b}_0(t_0)L),$$

i.e. stretching-out represents labour-augmenting technical change at the macro level. It is easy to see that $J(t_1) = J(t_0)$, i.e. stretching-out does not increase the total productive capacity of industry.

Obviously, the combination of the two cases also results in an invariant aggregate production function. The combination can be represented as follows.

$$(11) \quad \mathbf{f}(\mathbf{b}, t_1) = a/b \mathbf{f}[\mathbf{b}/b, t_0].$$

The aggregate production function (at $t = t_1$) corresponding to (11) can be written as

$$(12) \quad Q = F(aJ(t_0), b\mathbf{b}_0(t_0)L).$$

Both capital-augmenting and labour-augmenting technical change can therefore be represented as particular changes in the capacity density function. As (11) illustrates, the invariance requires that only certain kinds of shifts in \mathbf{f} keep F invariant. The basic form of the distribution should remain unchanged. This general condition is satisfied if the pace of factor-augmenting technical change is equal in each firm.

Without empirical research it is impossible to assess how stringent these invariance conditions are. It is important to recognize that, because of the differing roles of labour and capital, labour-augmenting and capital-augmenting technical change also have differing roles within Sato's framework. Capacity density functions are functions of labour efficiency coefficients only. Therefore analysis of the nature of labour-augmenting technical change becomes the crucial issue, and, consequently, examination of stretching-out patterns. Changes in \mathbf{f} can be analysed by examination of the stretching-out condition.

In what follows we therefore utilise the stretching-out condition when we analyse the nature of technological change in the Finnish manufacturing industries by utilising micro data. Even though invariance conditions in Sato's approach are closely related to the aggregation of production functions, they can be utilised in the characterisation of the nature of technological change. They provide a means of characterising the nature of technological change in a manner which we call the *parametric* way of characterising technological change. The basic classification is obvious. In an industry, technological change is *smooth*, if the shape of the industry-level aggregate production function remains unchanged. It is *non-smooth*, if the shape of the aggregate production function changes.

Smoothness of technological change in an industry also defines structural change. According to our definition, structural change takes place in an industry, if technological change is non-smooth i.e. the shape of the industry-level aggregate production function alters.

In his survey of productivity and structural change, Krüger (2008), by referring to the definition by Streissler (1982), defines structural change as ‘long-term changes in the composition of economic aggregates’. In our paper the economic aggregate is an industry and the ‘composition’ of the economic aggregate is characterised by the industry-level production function. It is important to recognize that, by our definition, technological change can be smooth even in the presence of entries and exits of firms or plants. They need not change the aggregative technological structure of an industry.

Even though smoothness is defined by the utilisation of the invariance of aggregate production functions, these functions need not be estimated when the aggregation theory outlined above is utilised. Only capacity density functions should be estimated. The invariance conditions can be analysed by examining the stability of the relevant density functions over time.

It is not obvious from which class of statistical distribution functions the capacity density functions should be chosen. By leaning on the vintage theory of capital, Sato constructed a model in which the distribution was an exponential curve. Yet, he himself thought that one should expect that the distribution should be bell-shaped in the majority of cases (Sato 1975 p. 197-198). The empirical evidence which was at Sato’s disposal also supported that kind of view (Sato 1975, Ch. 13 and 14). In the empirical analysis, Sato therefore utilised the class of beta distributions which include, among others, bell-shaped distributions. We follow Sato and also estimate beta distributions when we conduct our empirical analysis. In the Appendix, we discuss beta distributions further and outline our estimation strategy.

Ideally, one could utilise some theoretical models of structural change when choosing the class of distributions. Of those, the ones presented, for example, in Caballero and Hammour (1994, 1996) and Caballero (2007, Ch. 4) may be the most relevant. The models include distribution functions which are similar to the capacity density functions used in Sato’s approach. Even though the models are far from being simple (unlike the models Sato (1975) used over thirty years ago, they, for example, contain entries and exits of firms as a major factor giving rise to restructuring) they still seem to be too simple because the distribution functions they include are typically monotonic and not bell-shaped, which is also the shape consistent with the Finnish evidence. We continue this discussion in the concluding chapter.

3. DATA

The LMD data we use in this study is the Longitudinal Data on Plants in Manufacturing (LDPM). It has been constructed mainly for research purposes by Statistics Finland. The data is based on the Annual Industrial Surveys that, in principle, cover all manufacturing plants which employ at least five persons, up to 1994. Thereafter it has included all the plants employing at least 20 persons. In this paper we have taken into account this break by excluding plants with fewer than 20 persons.

Even though the LDPM data consists of annual data from 1975 until 2005 we use five-year windows and utilise the data from the years 1980, 1985, 1990, 1995, 2000 and 2005. For our purpose this is enough because we are not interested in short-run variations.

For our analysis, we need plant-level data on labour productivities or labour shares. (See the Appendix for a discussion of the alternative uses of these variables.) We utilise both variables mainly in order to check the robustness of the results. Output is measured by value added, and nominal measures are deflated by means of industry-specific producer price indexes. Labour input is measured by total hours worked. The labour share is defined as the share of the total labour costs in value added.

We have used the following industry classification, which is close to the standard two-digit industry classification: “Food” (NACE 15-16), “Textile” (17-19), “Wood” (20), “Paper” (21), “Printing” (22), “Chemicals” (23-25), “Minerals” (26), “Metal products” (27-28), “Machinery” (29 and 34-35), “Electrical equipment” (30-31), “Communications equipment” (32-33), and “Other (36-37). This classification enables us to pay special attention both to the traditional industries like the paper industry and also to the emerging industries like the manufacture of communications equipment.

4. DESCRIPTIVE ANALYSIS

During the past twenty-five years, the Finnish manufacturing industry has experienced a profound change. Traditionally, firms which have been operating in two industries, the forest industry and the metal industry, have produced most of the total production. Within the metal industry, the manufacture of machinery and equipment was the most important industry for many decades after the Second World War. A drastic change within the metal industry took place in the 1990s when the importance of the manufacture of communications equipment started to rise (Table 1).

Finland is one of those countries that have been strongly affected by the rise of ICT technology. In the international division of labour, Finland has been, with the Nokia Company as the major player, a prominent manufacturer of ICT technology.

Table 1. The structure of the Finnish manufacturing industry in 1980 and 2005.

Industry	1980	2005
Food	10.9	7.4
Textile	7.8	1.4
Wood	8.7	3.9
Paper	16.1	11.5
Printing	5.9	4.9
Chemicals	12.5	13.2
Minerals	3.9	4.0
Metal products	9.5	11.0
Machinery	15.6	15.9
Electrical equipment	4.0	3.6
Communications equipment	2.0	22.6
Other	3.2	1.5

Note: The figures depict percentage shares of total value added of the manufacturing industry.

Source: Authors' calculations based on the LDPM data.

For the Finnish economy, the 1990s was a special period at least for two further reasons. During the early 1990s, the Finnish economy experienced a very deep depression with GDP decreasing by 11 per cent. This gave rise to a strong restructuring which was characterised by bankruptcies and plant closures. Finnish membership of the European Union in 1995 has also been a major factor, especially influencing the developments of some traditional industries.

These factors, for example, are potential major causes of structural change in various industries. In what follows, we shall first have a closer look at the developments in the twelve major industries by considering stretching-out patterns in those industries during the period from 1980 until 2005, by utilising five-year windows.

For these industries, Figure 3 depicts the capacity density functions which have been estimated non-parametrically. Even though the density functions are estimated non-parametrically we can make some tentative characterisations about the nature of technological change for each industry. As expected, the capacity density function in each industry has stretched out since 1980. This means that, in each industry, labour productivity has grown. Yet there are major differences as to how capacity density functions have stretched out. In those industries in which average labour productivity has increased less than on average, the family of density functions tends to be less scattered than in those industries in which labour productivity has increased faster than on average. Not surprisingly, the most scattered families are found in the manufacture of electrical equipment (NACE 30-31) and in the manufacture of communications

equipment (NACE 32-33). In these rapidly expanding industries, the rate of average labour productivity growth has also been fastest.

However, the magnitude of average labour productivity growth does not necessarily indicate how smooth technological change has been in various industries. If labour productivity has grown at approximately equal speed in the plants of rapidly growing industries, rapid growth may be associated with smooth technological change. Analogously, slow average labour productivity growth may be associated with non-smooth technological change. Even though examination of the smoothness of technological change requires a more detailed statistical analysis of the stability of the density functions over time, something can be said just by only looking at the families of density functions in Figure 3.

Figure 3. Stretching-out patterns in the Finnish manufacturing industries 1980-2005.

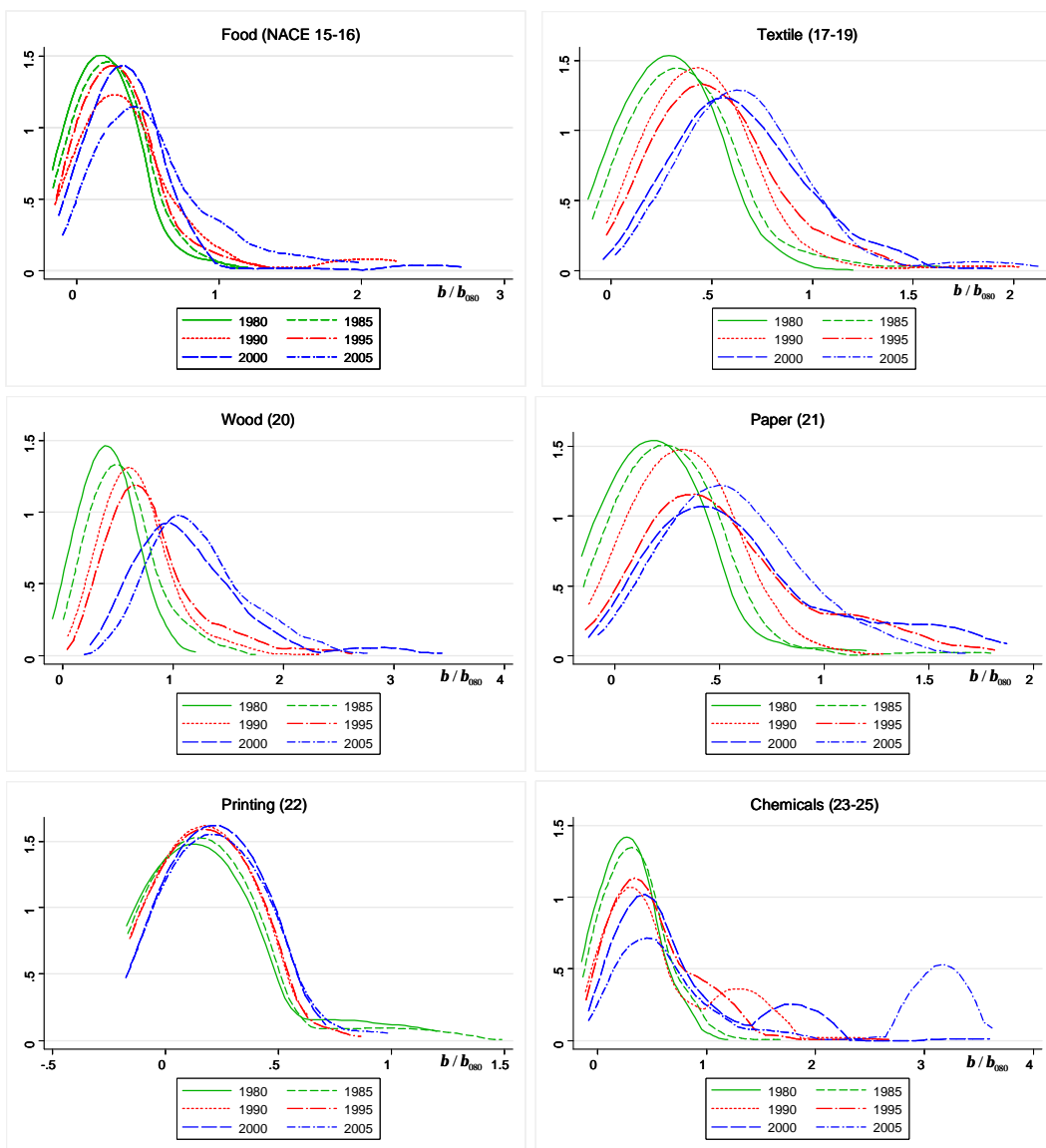
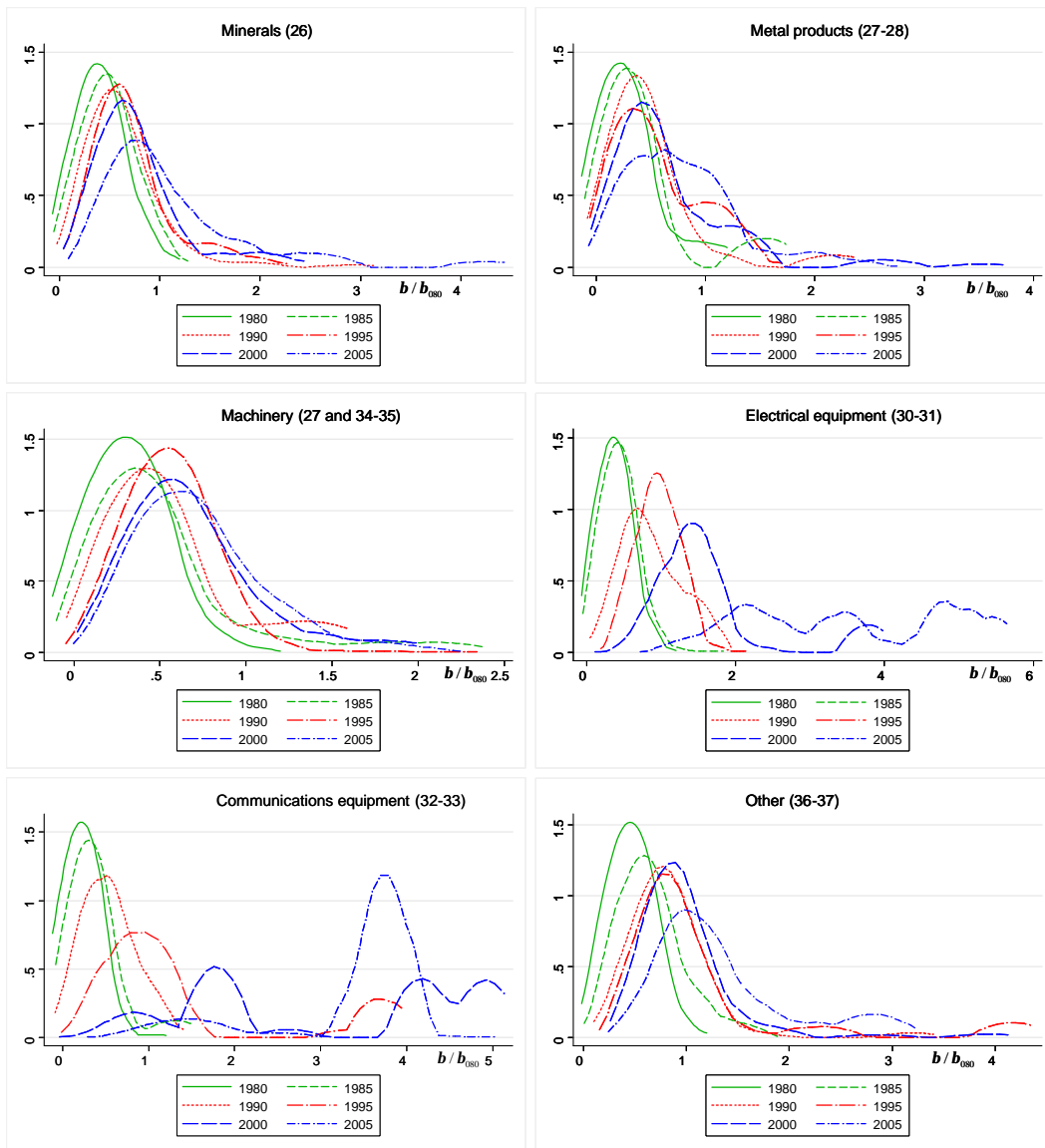


Figure 3 (continued)



Notes: The distributions, which describe the distribution of the total output in each industry, have been obtained by the use of kernel density estimations with output-weighted observations on relative labour productivities defined by b/b_0 . For each industry, b_{080} denotes the sample maximum of the level of labour productivity in 1980. Kernel density estimates are based on the use of the Epanechnikov kernel with bandwidth = 0.1. The bandwidth parameter was chosen with the intention to produce distributions which do not have several modes.

First, the early 1980s seem to be a period of relative tranquil developments in almost every industry. In each industry, the capacity density function stretches out with the peak of the distribution lowering in a way that is consistent with the pattern depicted by Figure 2a. It is noteworthy that when density functions are estimated non-parametrically, the functions are typically single-peaked, with some industries being, however, interesting exceptions.

Manufacture of food products seems to have been an industry of relatively smooth change, at least until the 1990s. Thereafter, the stretching-out pattern indicates the presence of structural change. Among the twelve industries, the Finnish paper industry seems to be an example of an industry of relative tranquil developments, at least in comparison to the metal industry. In each of the five branches of the metal industry, non-smooth developments have been characteristic, at least during some periods. The manufacture of electrical equipment and the manufacture of communication equipment seem to have been the most turbulent ones. In those industries, capacity density functions have even been multimodal in some years. It is interesting that, in the manufacture of communications equipment, the turbulence seems not to have started until the mid-1990s, which is when the growth of that industry really took off. This is reflected in the stretching-out pattern as a wide stretch in the density functions since the mid-1990s. The pattern also seems to indicate that a larger share of aggregate output in this industry has gone to high-productivity plants.

In order to link our empirical analysis closer to the theoretical framework outlined earlier, we next fit beta distributions to the data and analyse the stability of the estimated beta distributions over time. This is, of course, only one alternative approach if one wants to examine the stability of density functions over time. Within a more general framework one could, in principle, analyse stability non-parametrically without specifying the class of density functions in detail. However, the use of a parametric approach, like the one utilised by Sato, enables one to have a more detailed picture of the nature of possible changes in capacity density functions, and in industry-level production functions, too.

5. ECONOMETRIC ANALYSIS

Our estimations are examples of beta regressions. (See the Appendix for the discussion of beta distributions, beta regressions, and our estimation strategy). Beta regressions are useful, especially in the case where dependent variables are proportions (Paolino 2001, Ferrari and Cribari-Neto 2004, Smithson and Verkuilen 2006). In that case the use of regressions which are based on the normal-theory of (linear) regression may be questionable because the basic assumption of normality of errors is not fulfilled and the resulting estimates may be inaccurate.

Our use of the algorithm which utilises maximum-likelihood estimators allows us to estimate the shape parameters of a standard beta distribution. Because standard beta distributions are defined over the unit interval, we use scaled variables in the estimations. This means that the parameters \mathbf{b}_0 and s_0 (see (A.7) in the Appendix) are assumed to be known *a priori*, and therefore scaling is possible.

Ideally we would have theoretical maximum and minimum values for these parameters, but this is not the case. Therefore we use the maximum and minimum values provided by the data. In the absence of

theoretical maximum and minimum values, this may be the only possibility. (In principle, these scale endpoints could be estimated by the utilisation of other algorithms.) A problem with this choice is, for example, that measurement errors may have a strong influence on the results. Consequently, the results are likely to indicate non-smooth developments because measurement errors affect the estimates of shape parameters.

Despite these difficulties we believe that we are able to characterize various industries, for example, as industries with tranquil or turbulent developments. As mentioned earlier, invariance conditions are so stringent that one does not normally expect them to be fulfilled.

The estimations are based on the use of data on relative labour productivities and relative labour shares. (For a discussion of these variables, see the Appendix). The data on labour productivities for each plant in each industry is scaled by the use of the transformation

$$(13) \quad u_t = \frac{\mathbf{b}_t / \mathbf{b}_{0t} - \min(\mathbf{b}_t / \mathbf{b}_{0t})}{1 - \min(\mathbf{b}_t / \mathbf{b}_{0t})},$$

where \mathbf{b}_{0t} is the sample maximum of the level of labour productivity and $\min(\mathbf{b}_t / \mathbf{b}_{0t})$ the sample minimum of the relative labour productivity for each year $t, t = 1980, 1985, 1990, 1995, 2000, 2005$.³

Minimum values are used in order to define u_t such that $u_t \in (0,1)$.

Analogously, data on labour shares were scaled by the use of the transformation

$$(14) \quad y_t = \frac{s_{0t} / s_t - s_{0t}}{1 - s_{0t}},$$

where s_{0t} is the sample minimum of the labour share for each t .

The results are summarised in Figures 4 and 5. For each industry, the capacity density functions should be similar if the invariance condition is satisfied. Both figures give the same interpretation. The shape parameters of capacity density functions seem to be unstable over time. However, in some industries changes are more pronounced than in other industries. As in Figure 3, capacity density functions for the manufacture of electrical equipment and for the manufacture of communications equipment seem to exhibit strong alterations over time. On the other hand, in some industries, as in the manufacture of food products, changes have been less dramatic.

³ Subscripts for plants and industries have been omitted.

Yet, by only looking at the families of density functions in Figures 4 and 5, it is impossible to get a good understanding of the nature of changes in various industries. The basic reason is obvious. Even small changes in the values of the shape parameters can give rise to large changes in the density functions, because, for example, a small change in a parameter may transform a unimodal distribution into a monotonic curve. Therefore, one should have information about the statistical significance of possible changes in the shape parameters.

Figure 4. Capacity density functions for the Finnish manufacturing industries: relative labour productivities .

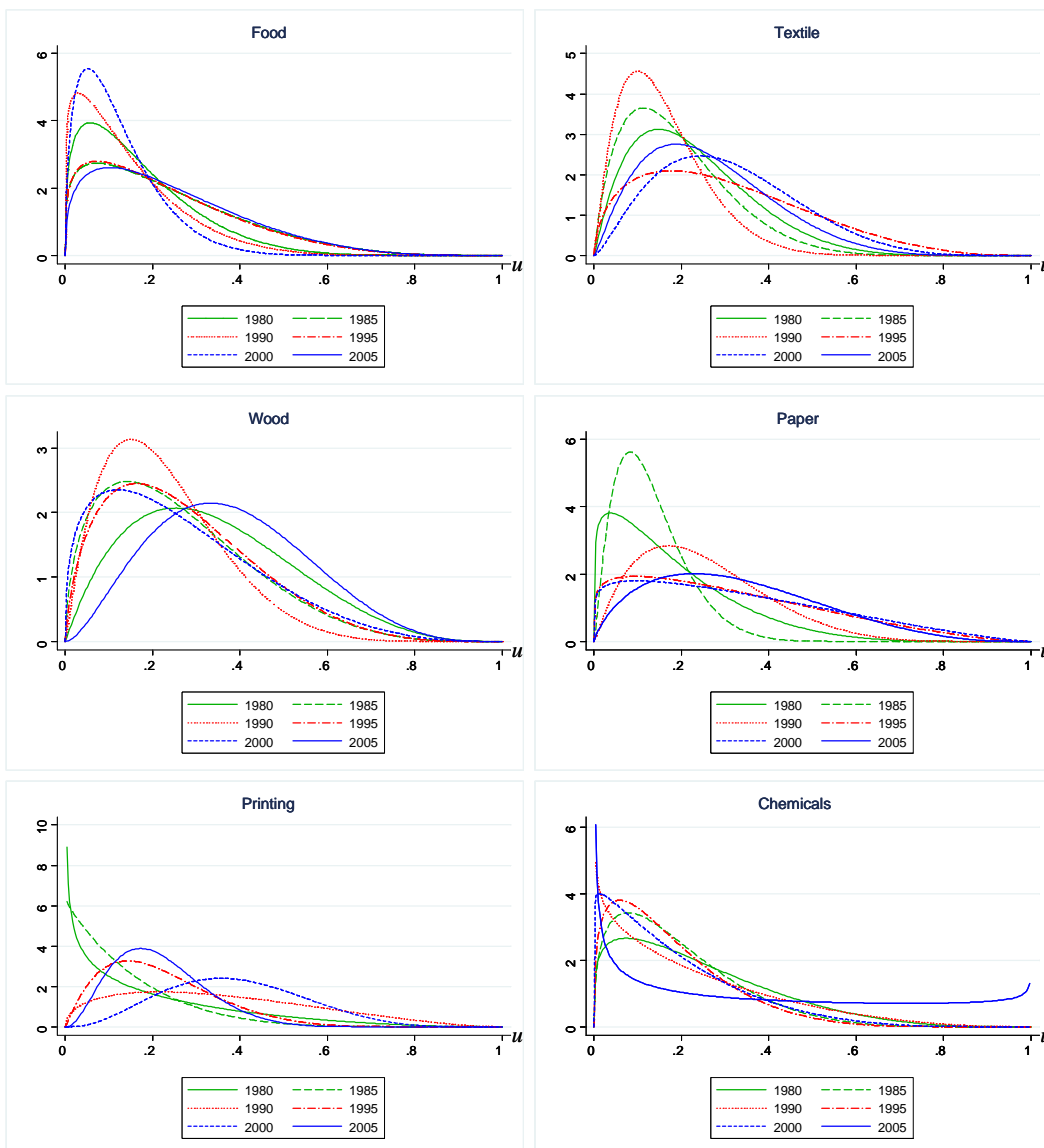
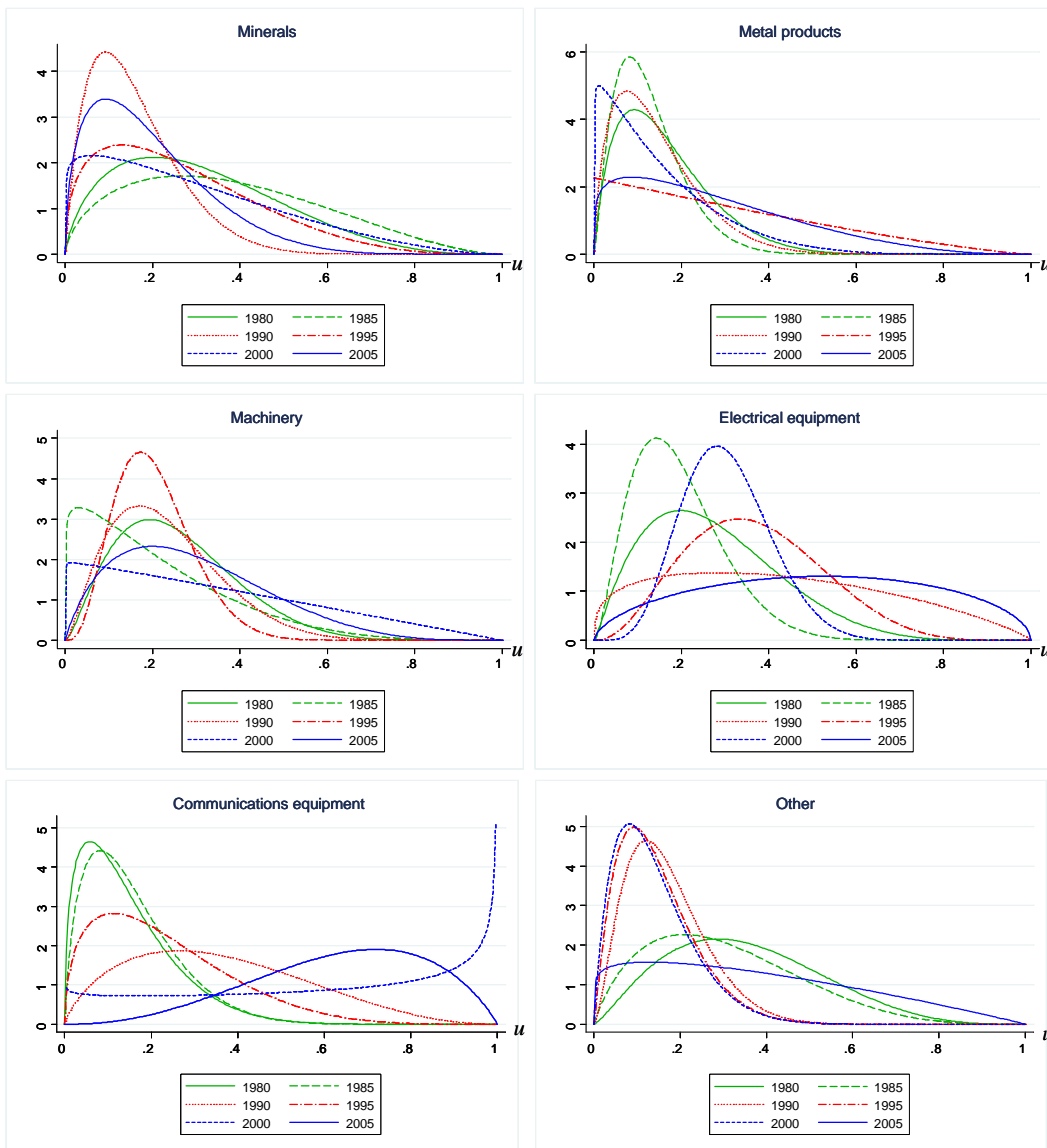


Figure 4 (continued)



Notes: u is defined as in equation (13). For each year, the estimates of the standard beta distributions have been obtained by the use of output-weighted observations and maximum likelihood estimators.

Figure 5. Capacity density functions for the Finnish manufacturing industries: relative labour shares.

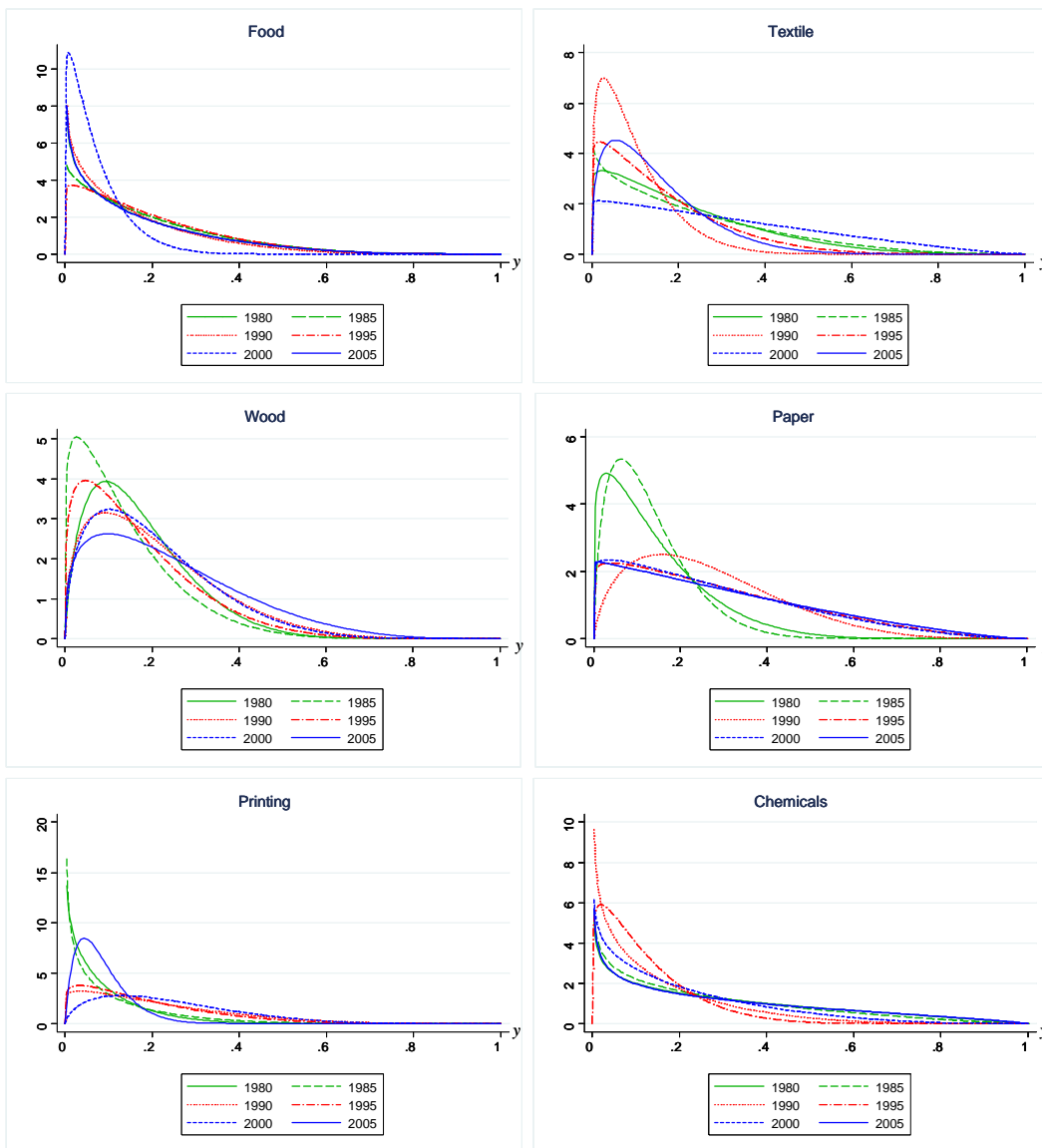
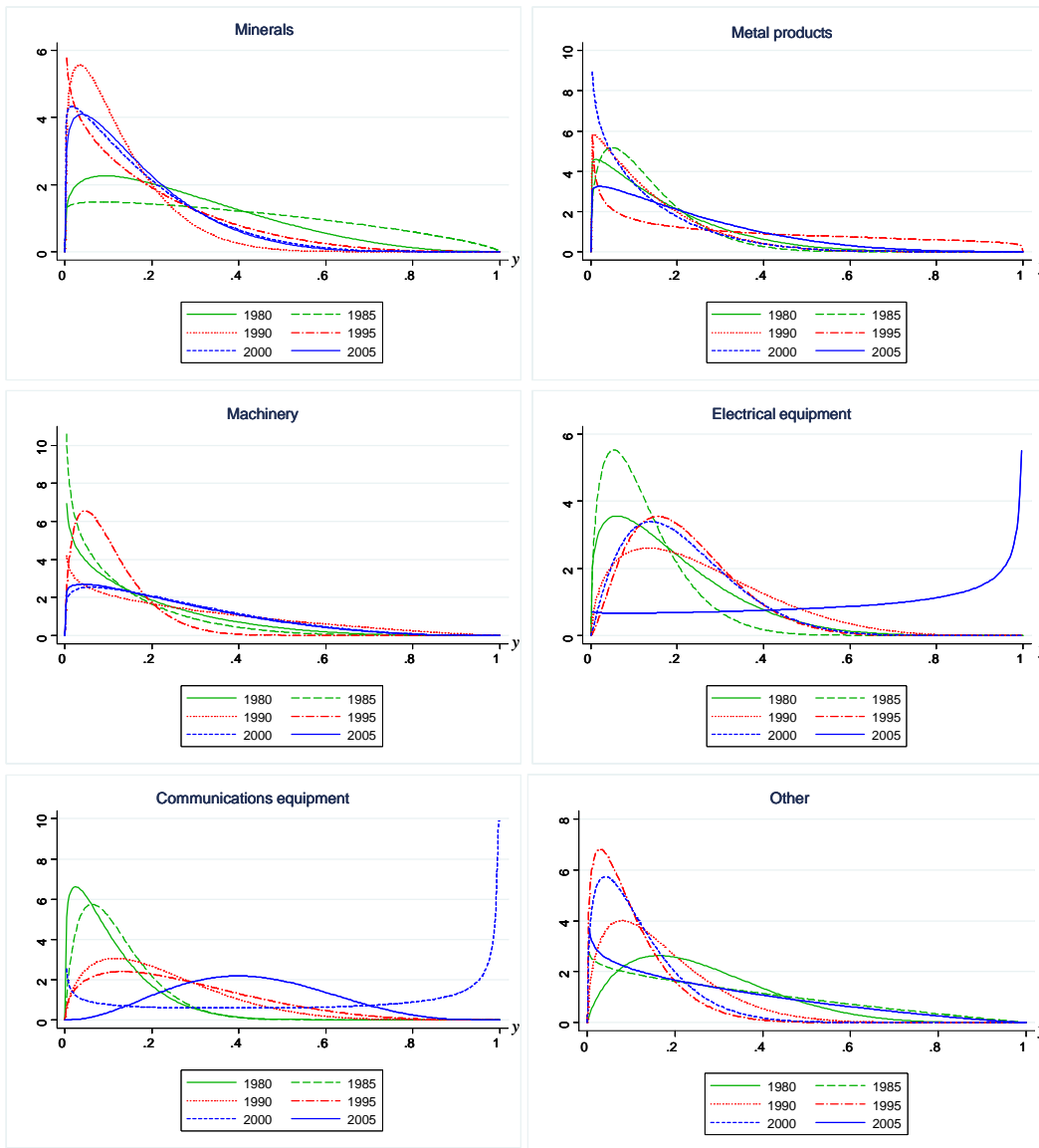


Figure 5 (continued)



Notes: y is defined as in equation (14). For each year, the estimates of the standard beta distributions have been obtained by the use of output-weighted observations and maximum likelihood estimators.

Within the framework of beta regressions, the stability of the parameters \mathbf{I} and \mathbf{n} can be analysed by modelling them separately. Furthermore, with reparameterization, \mathbf{I} and \mathbf{n} can be translated into location and dispersion parameters, which makes the interpretation of the nature of possible changes in capacity density functions easier. In the Appendix we have outlined the manner in which the constancy of the location and dispersion parameters, which we have denoted by μ and \mathbf{f} , has been analysed.

The results for the use of data on both relative labour productivities and relative labour shares are reported in Table 2 and Table 3, respectively. They are based on the utilisation of the whole data set and indicate whether structural change has taken place since 1980. The most important difference between the results in

Table 2 and Table 3 is that the results in Table 2 indicate more turbulent developments than the results in Table 3. According to the results of Table 2, non-smooth technological change is rather the rule than an exception. This result is not surprising because we have already remarked that the manner in which the variable of relative labour productivity is defined may give rise to some extra nonstability of the capacity density function over time. We therefore base our interpretations mainly on the results in Table 3.

Table 3 indicates that a similar common pattern of development cannot be distinguished among the twelve industries. In some industries development has been smoother than in others. Still, a smooth process of development seems to be an exception. This should not come as a surprise because our definition of smoothness is stringent.

The basic result, which is common for a relevant number of the industries, is that the early 1980s were a period of relatively tranquil overall development. Developments became more turbulent after the mid-1980s. This kind of characterisation is by no means new. It was one of the main findings of Maliranta's (2003) analysis of the micro-level dynamics of productivity growth in the Finnish manufacturing industries. Unlike this study, Maliranta (2003) was based on a variant of a well-known and widely used non-parametric method by which aggregate productivity growth can be decomposed into various micro-level sources (see Maliranta 2003). The turbulence has been reflected in different ways in various industries. In some industries the period of turbulence did not start earlier than in the 1990s (Maliranta 2005).

Table 2. Stability of capacity density functions in the Finnish manufacturing industries 1980-2005: relative labour productivities.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
<i>logit μ</i>												
constant	-1.59***	-1.22***	-0.67***	-1.53***	-1.40***	-1.16***	-0.80***	-1.64***	-1.06***	-0.98***		-0.61***
year85	0.41***	-0.22***	-0.36***	-0.36**	-0.42***	-0.28**	0.29**	-0.28***	-0.27***	-0.46**		-0.22*
year90	-0.18***	-0.44***	-0.55***	0.43***	0.80***	-0.10	-0.86***	-0.15**	-0.14*	0.66***		0.98***
year95	0.38***	0.39***	-0.32***	0.74***	0.13	-0.39***	-0.22+	0.82***	-0.30***	0.44***		-1.15***
year00	-0.37***	0.42***	-0.33***	0.83***	0.95***	-0.32***	-0.1	-0.08	0.38***	0.14		-1.19***
year05	0.46***	0.20**	0.18**	0.82***	0.11	0.77***	-0.62***	0.66***	0.19***	1.01***		0.07
<i>ln f</i>												
constant	2.07***	2.19***	1.77***	1.88***	1.24***	1.63***	1.70***	2.45***	2.27***	2.07***		1.89***
year85	-0.40***	0.10	0.02	0.95***	0.72***	0.37**	-0.24	0.46***	-0.60***	0.59**		-0.06
year90	0.02	0.41***	0.44**	0.23	0.17	-0.27	0.79***	0.07	0.12	-0.93***		0.87***
year95	-0.38***	-0.56***	0.07	-0.57***	1.01***	0.40**	-0.02	-1.26***	0.75***	0.14		0.81***
year00	0.46***	-0.14	-0.13	-0.64***	0.95***	0.09	-0.36*	-0.51***	-1.17***	0.96***		0.76***
year05	-0.34***	-0.08	0.18	-0.21	1.44***	-1.30***	0.37*	-0.96***	-0.42***	-0.93***		-0.76***
Observations	2100	1504	1393	662	1377	1384	920	2040	2713	560	546	891

Notes: I=Food, II=Textile, III=Wood, IV=Paper, V=Printing, VI=Chemicals, VII=Minerals, VIII=Metal products, IX=Machinery, X=Electrical equipment, XI=Communications equipment, XII=Other. Statistical significance: + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Estimates are results of beta regressions which are based on the use of output-weighted observations and on the utilisation of the link functions which are represented by (A.11) and (A.12). In the estimations, the year 1980 has been the benchmark year. For industry XI, no estimates are presented because in the estimation convergence was not achieved.

Table 3. Stability of capacity density functions in the Finnish manufacturing industries 1980-2005: relative labour shares

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
<i>logit μ</i>												
constant	-1.56***	-1.32***	-1.56***	-1.80***	-2.34***	-0.89***	-0.97***	-1.63***	-1.56***	-1.48***	-2.12***	-1.05***
year85	0.11	0.10	-0.26**	-0.12	0.19	-0.18	0.50***	-0.25*	-0.34***	-0.48**	0.14	0.25*
year90	-0.13	-0.87***	0.22**	0.77***	1.02***	-0.86***	-0.99***	-0.21*	0.59***	0.38*	0.84*	-0.55***
year95	0.14	-0.31***	-0.03	0.88***	0.84***	-1.07***	-0.49***	1.04***	-0.58***	0.18	1.11**	-1.12***
year00	-1.00***	0.52***	0.20**	0.84***	1.17***	-0.51***	-0.63***	-0.28**	0.46***	0.16	2.59***	-0.95***
year05	0.46	-0.42**	0.42***	0.94***	-0.04	-0.01	-0.64***	0.34***	0.43***	1.52***	1.82***	0.11
<i>ln f</i>												
constant	1.52***	1.62***	2.31***	2.12***	2.23***	0.95***	1.50***	1.86***	1.58***	1.95***	2.44***	1.95***
year85	0.09	-0.22*	-0.19	0.46+	-0.46**	0.13	-0.49*	0.54***	0.20	0.59*	0.25	-0.86***
year90	0.16	0.92***	-0.35**	-0.27	-0.57***	0.68***	0.83***	0.16	-0.46***	-0.11	-0.42	0.29+
year95	0.16	0.27*	-0.31*	-0.80***	-0.40**	1.27**	0.04	-1.28***	1.15***	0.49*	-0.73	0.60***
year00	1.21***	-0.43***	-0.26*	-0.78***	-0.31+	0.47**	0.35+	0.04	-0.05	0.35	-2.43***	0.50**
year05	-0.02	0.62***	-0.60***	-0.90***	-0.90***	-0.06	0.48**	-0.27+	-0.07	-1.49***	-0.40	-0.82***
Observations	2106	1510	1399	668	1383	1390	926	2046	2719	560	552	887

Note: See the note at Table 2.

The best example is the manufacture of food products. As the first column in Table 3 illustrates, this industry was not subject to major structural changes in the 1980s nor in the early 1990s. The period of turbulence started in the late 1990s. It is highly likely that this is related to membership of the European Union, which took place in 1995. Thereafter the manufacture of food products has been subject to enhanced foreign competition, which has had a strong influence on its development. Overall, the manufacture of food products has been an industry of relatively smooth technological change, however.

The Finnish paper industry may be the best example of a traditionally important industry which experienced a particular period of turbulence in the late 1980s. In the 1990s and later, developments have been relatively smooth, however. The shape of the capacity density function has been relatively stable since the mid-1990s. (It can be shown that the differences in the parameters for the years 1995, 2000 and 2005 in Table 3 are not statistically significant.)

On the other hand, developments in the manufacture of communications equipment were relatively tranquil until the late 1990s, when the ICT expansion took off. Among the other sub-industries in the metal industry, the manufacture of machinery and equipment is an example of more or less constant change.

Even though the definition of smooth technological change is stringent in the sense that the invariance condition is normally not satisfied, the estimations presented above illustrate that by using the definition we can find major differences between various industries.

The estimations were based on the use of total samples for each year. Nevertheless, we can have a closer look at how entries and exits affect the results. Does non-smooth development reflect the importance of entries and exits or is it an indication of non-smooth development among continuing plants?

According to Table 4 the period of turbulence is normally associated with turbulence among continuing plants. It illustrates whether, within a five-year period, non-smooth technological change is associated with continuing plants or not. For example, the start of the turbulence in the Finnish paper industry in the late 1980s is reflected as a change in the capacity density function of the continuing plants. Analogously, the rise of the Finnish ICT sector (Industry XI) was accompanied by an alteration in the shape of the capacity density function among the continuing plants. Furthermore, in the manufacture of machinery and equipment, which we regard as an example of an industry with more or less constant change, the capacity density function of the continuing plants has not remained unchanged.

It is straightforward to analyse the importance of entries and exits for the stability of the capacity density functions by using dummy variables identifying entering and exiting plants in the regression

equations. We will not report those beta regressions here. (They are available upon request.) The main finding confirms the above result. If an industry experiences a period of turbulent developments it is already reflected among continuing firms. Normally, entries and/or exits are not the sole causes of turbulence. This is in line with the earlier analysis performed by the non-parametric decompositions of micro-level sources of aggregate productivity growth which has shown that the entry and, especially, the exit component often vary hand in hand with the between component of the continuing plants (see Maliranta 2003, page 262). An obvious explanation for the co-movement of the components is that they all measure a slightly different aspect of the same underlying technological transformation taking place at the level of plants. Entries and exits are the “extreme forms” of the renewal mechanism. Both are time-consuming processes. Entries entail a lot of experimentation and selection. Similarly, typically exiting plants have experienced a sustained decline in their relative size and relative productivity many years before their exit (i.e. when they were incumbents).

Table 4. Stability of capacity density functions among continuing plants: relative labour shares for selected industries.

	I	IV	VIII	IX	X	XI
logit μ						
constant	-1.55***	-1.79***	-1.61***	-1.61***	-1.47***	-2.10***
years8085	0.11	-0.14	-0.25**	-0.42***	-0.47	0.14
$\ln f$						
constant	1.51***	2.09***	1.84***	1.64***	1.95***	2.46***
years8085	0.07	0.49	0.55***	0.33**	0.58**	0.36
N	847	221	506	804	151	108
logit μ						
constant	-1.45	-1.91***	-1.84***	-2.15***	-1.87***	-2.04***
years8590	-0.36***	0.90***	0.06	1.21***	0.80***	0.79***
$\ln f$						
constant	1.61***	2.58	2.39***	2.31***	2.32***	2.73***
years8590	0.28***	-0.72***	-0.32*	-1.22***	-0.71**	-0.79***
N	789	200	514	786	163	137
logit μ						
constant	-1.91***	-1.06***	-1.77***	-0.94***	-1.20***	-1.23***
years9095	0.54***	0.15	1.21***	-1.18***	-0.11***	0.27
$\ln f$						
constant	2.01***	1.95***	1.97***	1.06***	1.85***	2.05***
years9095	-0.39***	-0.63***	-1.42***	1.71***	0.71**	-0.43
N	553	207	414	580	140	134
logit μ						
constant	-1.47***	-0.90***	-1.47***	-2.13***	-1.26***	-0.98***
years9500	-1.07***	-0.07	-0.55***	1.08***	-0.07	1.53***
$\ln f$						
constant	1.82***	1.32***	0.57***	2.74***	2.51***	1.65***
years9500	0.89***	-0.07	1.59***	-1.22***	-0.12	-1.62***
N	443	199	464	610	131	163
logit μ						
constant	-2.56***	-0.96***	-1.86***	-1.10***	-1.28***	0.18
years0005	1.03***	0.10	0.69***	-0.06	1.30***	-0.46**
$\ln f$						
constant	2.70***	1.32***	1.90***	1.55***	-2.06***	0.22
years0005	-1.21***	-0.10	-0.34**	0.13	2.42***	1.87***
N	360	192	611	696	129	164

Note: Time dummies indicate the plants which were operating in the two years represented by the names of the variables. For example, the dummy “years 8085” points to those plants for the year 1985 which were already operating in 1980. In this case, the year 1980 is the benchmark year. The estimates associated with the dummy variables therefore indicate whether capacity density functions have changed among continuing firms during a five-year period.

6. CONCLUSIONS

In this paper we utilised the theory of the aggregation of production functions in combination with a longitudinal micro-level data set (LMD) in examining technological and structural change in the Finnish manufacturing industry. One of our aims was to show that, even in the case in which LMDs are the basic sources of information, the use of the concept of the aggregate production function may be useful. We employed the concept in the characterisation of the smoothness of technological change in twelve industries. We called technological change *smooth* in an industry if the shape of the industry-level aggregate production function remained unchanged over time and *non-smooth* if the shape altered. Smoothness of technological change in an industry also defines structural change. According to our definition, structural change takes place in an industry if technological change is non-smooth.

Even though the shapes of aggregate production functions tend to alter over time in most industries, there are differences in timing and in the degree of turbulence between the industries. The analysis confirms the result obtained earlier that in some industries, for example, in the paper industry, the late 1980s marked the beginning of a period of relatively strong structural change. The food industry and the manufacture of communications equipment are examples of industries in which the 1990s was a period of turbulence. Overall, the manner which we called the *parametric* way of characterising technological and structural change turned out to be useful.

We carried out the investigation by reviving an old approach to the theory of the aggregation of production functions. We named it Sato's approach after its most important developer, Kazuo Sato (1975). Although the investigation of the invariance of aggregate production functions can be analysed by the use of Sato's approach, some modern dynamic models of restructuring are likely to provide the best analytical frameworks (see, for example, Caballero and Hammour 1994, 1996 and Caballero 2007, Ch. 4). However, so far no theory of the aggregation of production functions has been developed within these models, even though the models contain, in principle, the elements which would make it possible. Obviously, this is an area in need for further investigation.

Within Sato's approach the critical micro-macro link is provided by the capacity density function which describes the distribution of production capacity in an industry as a function of labour efficiency. The invariance of the aggregate production functions is analysed by examining the stability of capacity density functions over time. In the estimation of the capacity density functions we followed Sato and assumed that capacity density functions belong to the class of beta distributions. The algorithm which we had at our disposal allowed us to estimate standard beta distributions, and, accordingly, to utilise beta regressions in examining the invariance of aggregate production functions. This is, of course, only one alternative. The robustness of the results could be analysed more thoroughly by the use of other algorithms, or by the utilization of the class of distributions other than

the class of beta distributions. Some further robustness checks could also be performed by the use of non-parametric tests.

In the absence of good theoretical reasons, the utilisation of the class of beta distributions is an *ad hoc* choice. This gives another reason for the development of theoretical models which could be employed in the empirical analysis of structural change. The models that we especially have in our minds are dynamic statistical equilibrium models. Within those models a steady state or stationary capacity density function would be one part of a steady state solution of a model. That distribution might belong to the class of beta distributions.

In their survey of the use of LMDs in studies on productivity, Bartelsman and Doms (2000) ascribed the popularity of this research in part to the displeasure of the concept of the aggregate production function. In this paper we have attempted to show that, if this kind of research is firmly based on the traditional neoclassical theory of production or on some modern theories of restructuring, the concept of the aggregate production function may turn out to be useful. The theory of the aggregation of production functions can be utilised in order to construct the micro-macro link. However, there is need for the development of both the theory of the aggregation of production functions and modern theories of restructuring. In principle, these developments could be united.

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APPENDIX

Beta distributions and estimation of capacity density functions

If the capacity density function is assumed to be a beta distribution it can be written as

$$(A.1) \quad f(\mathbf{b} / \mathbf{b}_0) = (1/B)(\mathbf{b} / \mathbf{b}_0)^{\mathbf{l}} (1 - \mathbf{b} / \mathbf{b}_0)^{\mathbf{n}}, \mathbf{l} > -1, \mathbf{n} > -1, B > 0, 0 \leq \mathbf{b} / \mathbf{b}_0 \leq 1 .$$

In (A.1) labour efficiency coefficients have been scaled by \mathbf{b}_0 , and, accordingly, the distribution is defined over the unit interval. The distribution (A.1) is an example of the standard beta distribution. Scaling is useful when, for example, the invariance conditions are analysed and the distributions are estimated. B is a positive constant which is related to J by

$$(A.2) \quad \frac{1}{B} = \frac{J}{\mathbf{b}_0 B(\mathbf{l} + 1, \mathbf{n} + 1)},$$

where $B(\mathbf{l} + 1, \mathbf{n} + 1)$ is the beta function.

The class of beta distributions includes a wide variety of density functions. The two parameters \mathbf{l} and \mathbf{n} determine the shape of a distribution. Figures 1a-2b depict some alternative distributions of the beta type.

Because beta distributions fulfil the regularity conditions imposed by the existence condition, they generate a wide class of aggregate production functions. In general, these aggregate production functions do not belong to the class of CES production functions but to the class of VES production functions. Sato (1975, Ch. 17) analyses more carefully the properties of aggregate production functions which are generated by capacity density functions of the beta type. The analysis is far from straightforward because normally it is impossible to find an explicit solution to the aggregation problem. Only in some special cases (\mathbf{l} or \mathbf{n} are zero or positive integers) can integration be performed and an explicit solution obtained.

For us this is not a major problem, because we are only interested in the invariance of an aggregate production function whatever shape the function may take. For our purpose, estimation of capacity density functions is enough.

If the capacity density function is of the beta type, the stretching-out condition can be represented as follows. Let the capacity density function at $t = t_0$ be

$$(A.3) \quad f(\mathbf{b} / \mathbf{b}_0, t_0) = \frac{1}{B} \left(\frac{\mathbf{b}(t_0)}{\mathbf{b}_0(t_0)} \right)^l \left(1 - \frac{\mathbf{b}(t_0)}{\mathbf{b}_0(t_0)} \right)^n, l > -1, n > -1, B > 0,$$

where $\frac{\mathbf{b}(t_0)}{\mathbf{b}_0(t_0)} \in [0,1]$. Assume that at $t = t_1$ $\mathbf{b}(t_1) = b \mathbf{b}(t_0), b > 1$. This implies that

$$(A.4) \quad f\left(\frac{\mathbf{b}(t_1)}{\mathbf{b}_0(t_1)}\right) = f\left(\frac{\mathbf{b}(t_0)}{\mathbf{b}_0(t_0)}\right),$$

$$\frac{\mathbf{b}(t_1)}{\mathbf{b}_0(t_1)} = \frac{\mathbf{b}(t_0)}{\mathbf{b}_0(t_0)} \in [0,1].$$

By the use of $\frac{\mathbf{b}(t_1)}{\mathbf{b}_0(t_0)}$, the stretching-out condition can be represented as follows.

$$(A.5) \quad f(\mathbf{b} / \mathbf{b}_0, t_1) = \frac{1}{b} f\left(\frac{\mathbf{b}(t_1)}{b \mathbf{b}_0(t_0)}\right) = \frac{1}{b} \frac{1}{B} \left(\frac{\mathbf{b}(t_1)}{b \mathbf{b}_0(t_0)} \right)^l \left(1 - \frac{\mathbf{b}(t_1)}{b \mathbf{b}_0(t_0)} \right)^n,$$

where $b = \mathbf{b}_0(t_1) / \mathbf{b}_0(t_0) > 1$ and $\frac{\mathbf{b}(t_1)}{\mathbf{b}_0(t_0)} \in [0, b]$.

According to (A.3) and (A.5), examining invariance is, in principle, straightforward. Utilise data on labour efficiency coefficients and estimate the relevant parameters l and n at two points of time by making proper assumptions about b and $\mathbf{b}_0(t_0)$ (or $\mathbf{b}_0(t_0)$ and $\mathbf{b}_0(t_1)$). If the estimates are equal, invariance holds.

Unfortunately, the estimation is not so simple. A most important problem is the availability of data. Normally, data on labour efficiency coefficients is not available. Yet, if data on firm (or plant) level labour productivity and/or labour shares is available, the estimation becomes possible by making certain assumptions about micro-production functions.

If the availability of data on labour productivity is accompanied by the assumption that micro-production functions are of the Leontief type, data on labour productivity also provides data on labour efficiency coefficients. With this assumption about the micro-level production technology, data on labour shares can also be utilised in the estimation of capacity density functions. Furthermore, if micro data on labour shares is available, capacity density functions can also be estimated in more general cases in which micro-production functions are CES functions with $0 < s < 1$ (Sato 1975, Ch. 14).

In our study we assume that micro-production functions are of the Leontief type. This allows us to utilise plant-level data both on labour productivity and on labour shares and to estimate capacity density functions in two alternative ways.

If data on labour productivity is used, we estimate capacity density functions of the type (A.1) at two points of time and compare the estimates of \mathbf{l} and \mathbf{n} by choosing some values for $\mathbf{b}_0(t_0)$ and $\mathbf{b}_0(t_1)$ before the estimation.

If it is assumed that micro-production functions are Leontief type fixed proportions functions, the data on labour shares may be invaluable, especially in the case in which data on labour productivity is not available. Denoting the labour share by s and utilising the notation introduced earlier we get

$$(A.6) \quad s = xl/q = x/\mathbf{b}.$$

The lowest wage share is

$$(A.7) \quad s_0 = x/\mathbf{b}_0,$$

and consequently

$$(A.8) \quad s_0/s = \mathbf{b}/\mathbf{b}_0.$$

If, for example, the parameters of capacity density function (A.3) are to be estimated, data on s_0/s can be employed. The use of (A.8) utilises data on the inverse of the labour share. Equivalently, one can estimate the relevant parameters of the beta distribution by utilising the transformation $\mathbf{x} = \mathbf{b}_0/\mathbf{b} = s/s_0$ i.e. the inverse of the argument in, for example, density function (A.3). The use of the elementary theory of transformations of random variables gives

$$(A.9) \quad \mathbf{y}(\mathbf{x}) = \frac{1}{B} \mathbf{x}^{-l-n-2} (\mathbf{x}-1)^n, \quad l > -1, n > -1, B > 0,$$

where $\mathbf{x} \in [1, 1/s_0]$. In his estimations, Sato employs distribution (A.9) (Sato 1975, 200-201). He calls it the labour efficiency distribution.

We also estimate capacity density functions by utilising data on labour shares. Unlike Sato we base our analysis on the utilisation of (A.1) and (A.8). In examining the invariance condition we proceed in a similar fashion as in the utilisation of data on labour productivity. However, if labour share data is employed in the estimations, the characterisation of the invariance of a production function becomes

more complicated than in the case in which data on labour productivity is assumed to be available. This is due to the fact that real wage rate changes also affect labour shares.

In the paper, we stick to the basic assumptions about the competitive labour and product markets and assume that in each plant real wages grow at equal pace.⁴ This means that if developments in labour shares differ among firms, these differences are assumed to reflect changes in efficiency parameters. Obviously, the assumption is necessary if the data on labour shares is the only source of information.

By using that assumption, we can proceed as in the case in which data on labour productivities is available. Because $s_0 / s \in [s_0, 1]$ we can define variable y such that

$$(A.10) \quad y = \frac{\frac{s_0}{s} - s_0}{1 - s_0},$$

$y \in [0, 1]$. As before, we can assume that the distribution of y is a standard beta distribution.

The stability of the capacity density function over time is analysed by examining the stability of the distribution of y .

Beta regressions and the stability of capacity density functions

The validity of the stretching-out condition can be analysed by examining the constancy of the shape parameters \mathbf{l} and \mathbf{n} . Furthermore, with reparameterization these can be translated into location and dispersion parameters, which makes the interpretation of the nature of possible changes in capacity density functions easier. (See, for example, Smithson and Verkuilen 2006.)

If a random variable Y has a beta distribution with parameters \mathbf{l} and \mathbf{n} , then

$$E(Y) = \frac{\mathbf{l}}{\mathbf{l} + \mathbf{n}} \quad \text{and} \quad \text{Var}(Y) = \frac{\mathbf{l}\mathbf{n}}{(\mathbf{l} + \mathbf{n})^2(\mathbf{l} + \mathbf{n} + 1)}.$$

The use of notation $\mathbf{m} = E(Y)$, $\mathbf{s}^2 = \text{Var}(Y)$, $\mathbf{f} = \mathbf{l} + \mathbf{n}$ gives

$$\mathbf{s}^2 = \frac{\mathbf{m}(1 - \mathbf{m})}{(1 + \mathbf{f})}.$$

Reparameterization provides a location parameter μ and a parameter \mathbf{f} , which can be called a precision or dispersion parameter, because the variance decreases as \mathbf{f} increases. The variance also depends on the location parameter. It is easily seen that parameters μ and \mathbf{f} (like \mathbf{l} and \mathbf{n}) can be estimated separately by using maximum likelihood with the density function of the standard beta distribution defining the shape of the likelihood function. Furthermore, if parameters μ and \mathbf{f} are assumed to depend on independent covariates, the approach of an extended generalized linear model can be utilised in the estimation. Within that framework two link functions, which are usually nonlinear, smooth and monotonic functions, can be utilised in modelling μ and \mathbf{f} . (See, for example, Smithson and Verkuilen 2006.)

For each industry i our estimations provide parameters for the link functions which include time dummies as explanatory variables and are of the form

$$(A.11) \quad \ln(\mathbf{m}_i / (1 - \mathbf{m}_i)) = \mathbf{g}_{0i} + \mathbf{g}_{1i} \text{year85} + \mathbf{g}_{2i} \text{year90} + \mathbf{g}_{3i} \text{year95} + \mathbf{g}_{4i} \text{year00} + \mathbf{g}_{5i} \text{year05}$$

and

$$(A.12) \quad \ln(\mathbf{f}_i) = \mathbf{h}_{0i} + \mathbf{h}_{1i} \text{year85} + \mathbf{h}_{2i} \text{year90} + \mathbf{h}_{3i} \text{year95} + \mathbf{h}_{4i} \text{year00} + \mathbf{h}_{5i} \text{year05} .$$

In equation (A.11) a logit function has been used as a relevant link function, because for a beta-distributed dependent variable the mean has to lie in the open unit interval. In equation (A.12), the log function has been used, because the precision parameter \mathbf{f} has to be positive. (For a more thorough presentation of extended generalized linear models, see, for example, Smithson and Verkuilen 2006.)

The estimates indicate whether the capacity density functions changed after the year 1980. The results are given in Table 2 and Table 3. Similar considerations can be performed by the use of a year other than the year 1980 as the benchmark year. The results of modified versions of these equations are provided in Table 4. Obviously, there are also other alternatives for testing to see whether two capacity density functions for two different years are identical. However, if one assumes that capacity density functions belong to the class of beta distributions, the use of beta regressions suggests itself.

⁴ For at least some empirical support to this conjecture see Maliranta (2003)