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182

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Abstract

This study argues that firms can use profit sharing to raise profits by reducing the risk for inefficient separations by the workers. It is based on the idea of Hall and Lazear (1984) and clarifies their analysis of fixed wage contracts in employment relations of fixed length. It extends their analysis to continuing employment relationships in which the base wage is exogenously given but the firm can unilaterally set a profit sharing parameter. It is argued that the use of down payments, on which the analysis of Hall and Lazear is based, is not possible with continuing employment relationships. Instead the bargaining power will be reflected in the base wage. This reduces the possibilities to adjust the base wage to reduce inefficient separations and may increase the importance of profit sharing. The intuitive positive dependence of the profit sharing parameter on the covariance between the value of the worker's outside option and his productivity in the firm does not necessarily hold.

1 Introduction

A characteristic of remuneration systems that has become much more frequent in many countries in recent decades is that the employees' pay partly depends on some measure of the collective output of the employees or the value of it (OECD 1995, Alho 1998, Conyon and Freeman 2001). There are two main groups of explanations to the firms' use of profit sharing schemes. The first group consists of explanations relating to incentive effects. The second group of explanations takes the productivity for given and argues that profit sharing enables a more efficient level of employment or reduces inefficient separations. This study will concentrate on building a more solid ground for the second type of explanations.

Weitzman (1984, 1985) introduced the idea that profit sharing reduces the cost of labour and raises employment. The argument was that profit sharing presses down the base wage and this makes it cheaper for the firm to employ new workers. Later it has been shown that when firms take into account the full cost of employing workers, there is no increase in employment (Estrin et al. 1987, Layard et al. 1991).

However, in this study it is argued that downward rigidity in the base wage and uncertainty about productivity may lead to profit sharing having a positive effect on the efficiency of employment. This might also affect the level of employment. There may be several explanations to the existence of downward rigidity but the most obvious is that it may be necessary to agree on a wage cut with the employees. To credibly signal the need for a wage cut it may be necessary for the firm to take a costly conflict. A number of empirical studies also indicate that wage rigidity exists (Blinder and Choi 1990, Agell and Lundborg 1995, Bewley 1995, Campbell III and Kamlani 1997).

Because profit sharing conditions the pay of the worker on the profit of the firm, it adjusts the pay of the employees in accordance with the firm's cost of vacancies or possibly the value of the workers' outside option. Such flexibility will be useful, if there is downward rigidity in base wages and there is a need to raise wages, at least temporarily, to attract workers or reduce the costs of inefficient separations of incumbent workers. Hall and Lazear (1984) argued that when there is asymmetric information a fixed wage contract with no renegotiation is often more efficient than renegotiation. However, the cost is that there will be excess quits and layoffs. As Hall and Lazear pointed out, indexing the pay to a variable which is correlated with the workers' value to the firm (or the workers' payoff in their outside option) can reduce the costs of inefficient separations, without raising renegotiation costs which the fixed wage contracts were intended to avoid. This will be especially helpful if the productivity in the firm and the value of the outside option are correlated.

A similar point was made by Oyer (2000), who presented a model in which profit sharing is used to reduce inefficient separations. However, in his model the cost imposed on the firm by a separation is independent of profitability, and the outside option of the worker works as a signal to the firm about its profitability rather than the opposite way around. In a study of contract durations Azfar (2000) shows that performance related pay can reduce the need for other wage adjustments and lengthen contract duration. However, he does not provide any detailed analysis of the optimal payment scheme.

Profit sharing affects separations and employment by making the total expected wage more flexible. When there is downward rigidity in the base wage, the firms might be unwilling to raise it because there may be a need for lower pay in the future and downward rigidity makes the rise irreversible. In such a situation, adding a wage component that is conditional on profit raises pay if the productivity of the worker is high but also automatically reduces pay if profit falls. This makes it profitable for the firm to keep the employee in a downturn. This flexibility makes the firm more willing to raise the total pay of the employees when productivity is high. This may enable the firm to avoid inefficient separations and raise profit. In addition, the increased flexibility may induce the firm to employ more workers.

In the model in this paper, profit is for simplicity assumed to coincide with the worker's productivity. Therefore, I will use the term profit sharing for the conditioning of the worker's pay on the variable measuring the contribution to the surplus of the firm, although profit in reality is only one of many variables to which the firm may connect the pay of the employees. Because they are all correlated with the surplus of the employee in the firm, these other variables work in a similar way and the analysis also applies to such arrangements. Examples of such variables are: 1) the productivity of the employees in the firm or some part of it, 2) how much the firm or a team sells, and 3) the performance of a team concerning the quality of its output. Binding pay to productivity of the individual employee or the value of the individual's output would yield the best correlation between pay and contribution to output. Piece rates thus imply not only improved incentives but also higher correlation between the value of the employee to the firm and the pay. However, the value to the firm of keeping the employee does not only depend on the amount of output of the employee but also on the value of a unit of output. Then indexing pay to profit can be helpful as a complement to piece rates, because indexation makes pay depend on the price of the product.

Measurement costs often prevent the use of piece rates. Team work and interactions between employees can even make it impossible to measure the individual output. Variables concerning the collective performance are more easily measured and administered. Profit sharing or other collective remuneration schemes which make pay more flexible may therefore be feasible when measurement costs prevent the use of piece rates. In addition, there may be a greater need for such schemes in the absence of piece rates.

For indexation to be useful, it is necessary that the firm can credibly commit to follow it after the employee has decided to stay in the firm. If the outcome is verifiable and contracts can be enforced by a third party, this commitment is easily achieved. If it is not enforceable by a third party, it is equivalent to an implicit contract and the firm has to thrust on its reputation. Credibility is greatly improved if the conditioning variable is verifiable so that the employee can observe it and the firm is unable to manipulate it. Otherwise, the firm can reduce the value of the indexing variable once the work is done and, in consequence, reduce the pay of the employee below what it should have been. Although the firm would not act in such a way, the credibility problem reduces the payoff the employees expect and, in consequence, the effectiveness of indexing pay to the variable in question. However, if the firm has a high credibility because of a good track record concerning its handling of issues relating to the employees and conditioning of pay, verifiability becomes less important and an explicit indexing may not even be necessary.

The study does not claim that reduction of inefficient separations is the only reason for the use of profit sharing. Inefficient separations are likely to occur when the outside options for the parties are rather good but they are likely to be more costly when the outside option of one party is very poor. However, the model with a fixed wage with no renegotiation is less likely to apply when the expected loss due to separations is large, because sticking to a policy with no renegotiation could cause too large losses for the parties. In such a case there is likely to be costly renegotiations which profit sharing can help to avoid in an equivalent way. To explain such phenomena a model with incentive effects may be more relevant.

The empirical evidence on why firms use profit sharing schemes and other collective rewards has concentrated on incentive effects (OECD 1995). However, Himmelberg and Hubbard (n.d.) show that the rise in the pay of managers of large firms seems to be more related to inducing the managers to stay than to provide incentives. Moreover, according to the results of Azfar (2000) performance related pay is also associated with longer contract durations. These studies give some support to the notion that profit sharing can be used to reduce the risk for inefficient separations. Empirical studies of rent sharing also indicate that employees' pay is positively correlated with the firm's profitability, even when there is no explicit connection between the profit of the firm and the pay of the employees, but the reasons for profitable firms to pay high wages is unclear (Bronars and Famulari 2001, Abowd et al. 1999, Blanchflower et al. 1996). Suggested explanations to rent sharing include the notion that more profitable firms have a greater need to fill vacancies or that they have a greater need for loyal workers, because the marginal productivity of labour is higher in these firms. More profitable firms thus have more use for efficiency wages. Profit sharing also serves to raise the pay of the employee and it may therefore serve as a substitute for increases in the fixed part of the pay. Profit sharing may thus, at least partly, have the same explanations as rent-sharing in general.

The aim of this study is to provide a more thorough investigation of different contracts than in Hall and Lazear and extend the analysis to the case of profit sharing with an exogenous base wage. In particular, the study examines how different circumstances affect the share of the profits the firm pays to its labour force. The next section presents a variant of the model of Hall and Lazear (1984) and examines which contract type maximises surplus under various circumstances. Section 3 extends the analysis to profit sharing with an exogenous base wage. It is shown that profit sharing can serve to reduce the number inefficient separations and raise the profit of the firm, if the base wage is rigid. It is also shown that nonlinear schemes can be more efficient than simpler linear ones and that the optimal level of profit sharing depends on the probability distribution of the productivity and the value of the outside option in a complicated way. Section 4 provides a discussion of applications and extensions of the results. Section 5 concludes.

2 A fixed wage model

Firms and employees often face situations in which there is a bilateral monopoly, implying that there is a surplus for the parties to share.[1] Reasons for such bilateral monopolies include firm-specific

[1] The total surplus of the firm does not necessarily have to be positive. Because of non-linearities associated with e.g. fixed costs and team-work, the dismissal of any employee leads to a larger fall in output than in the wage costs of the firm. To continue to produce is nevertheless the best choice of the firm and the bilateral monopoly characterisation still applies.

human capital, search costs, and other costs associated with moving from one firm to another.

Let M denote the expected decline in production, which a separation of the worker from the firm leads to. Let A denote the employee's payoff in the outside option, which is an employment at a fixed wage in some other firm. At the time for the separation decision the firm knows only M and the worker knows only A. Thus, the setup follows Hall and Lazear (1984).

Because of variation in the value of the employee's outside option, the employee's reservation wage varies. Similarly, the productivity of the employee in the firm and the firm's profit from keeping the employee vary due to (at least partly) exogenous circumstances such as the demand for the products of the firm. This has to be taken into account when contracts are offered. Hall and Lazear (1984) argue that the asymmetric information of firms and their employees leads to an excessive number of quits and layoffs. Because of asymmetries in information, it would be costly to renegotiate contracts and the parties often prefer to write contracts in advance or give take-it-or-leave-it offers. According to their analysis, who gives the offer and how the contract is structured in optimum depends on the probability distribution of the surplus and the outside options as well as the possibilities of the parties to acquire information. As Hall and Lazear point out, it will usually be impossible to use information which is not available to both parties at a low cost. They argue that fixed pay may be a simple way of avoiding credibility problems and negotiation costs, but at the cost of inefficient separations. They also point out that if there are variables which are correlated with the outside options of the parties, it might be possible to write contracts which condition the pay on such variables and reduce the number of inefficient separations.

For tractability, I will assume a dichotomic distribution of M. This simplification compared to Hall and Lazear enables a deeper study of the extent to which profit sharing should be used by the firm to maximise profit. The assumptions concerning the outside options and the value of keeping the worker are as follows.

Assumption 1 *The value of having the worker in the firm, M, is discretely distributed and takes a high value M_h with probability $p(M_h)$ and a low value M_s with the probability $p(M_s) = 1 - p(M_h)$. The firm's outside option is normalised to 0. The worker's outside option, A, which may be correlated with M, is continuously distributed with a probability density function $g(A)$.*

These assumptions mean that if there is correlation between A and M so that $g(A|M_h)$ differs from $g(A|M_s)$, the parties also get more information on the value of the other variable when the value of A or M is revealed to them. Note that $g(A) = p(M_h)g(A|M_h) + p(M_s)g(A|M_s)$. The normalisation of the firm's outside option to zero means that M is the surplus of the firm net of the surplus in its outside option.

The employee and the firm are risk neutral and only care for the expected wage and profit respectively. The first best solution (as in Hall and Lazear) maximises

$$E(X(M, A)(M - A) + A), \tag{1}$$

in which X is a dummy function which takes the value 1 if $M \geq A$, and the value 0 otherwise. The employee should stay if and only if $M \geq A$, in which case $X = 1$. Denoting surplus by S the expected first best surplus is

$$E(\hat{S}^*) = \sum_{i=h,s} p(M_i)\Big(p(A < M_i)M_i + p(A > M_i)\int_{M_i}^{\infty} Ag(A|M_i)dA\Big). \tag{2}$$

As Hall and Lazear pointed out, insufficient information about A and M renders the first best solution impossible, if there is no (inexpensive) way for the parties to credibly transfer information to each other, which would be necessary to determine whether a separation is efficient. To be able to determine whether to separate, the parties at some point have to agree on a compensation scheme, which determines the pay of the employee. In this section the compensation scheme is the simplest possible.

Assumption 2 *The contract specifies a fixed wage w for a time period of length 1.*

To reach a second best solution and avoid negotiation costs or the cost of acquiring information, the parties specify a contract with a wage w that is sometimes outside the interval $[A, M]$. The process through which a contract is created affects the pay agreed on in the contract. Following Hall and Lazear I examine three kinds of contracts which differ with respect to the way in which the wage is set.

Assumption 3 *The wage setting process can be of three types:*
1. The contract can be signed in advance when only the probability distributions of the outcomes are known (called a contract of type p).
2. The firm sets the wage when it knows M (contract of type f).
3. The employee sets the wage when he knows A (contract of type e).

To denote the contract type which the wage w or surplus S originated from, the contract type will be used as an index. The distribution of rents depends on the negotiation power of the parties but the parties may use down payments so that the distribution of rents will in this section not affect the wage in the contract or probability for separation. Such payments include any explicit or implicit payment or binding agreement of it between the parties, such as advance pay or subsidy, before the outcomes concerning M and A are known. If there has been some (employment) relation between the parties before the period which the contract applies to, this also includes the possibility of paying over or below the market value for goods supplied during this period. Because the order in which the events occur is of great importance for the analysis, the following assumption is made.

Assumption 4 *The sequence of events is as follows:*
1. Both parties learn the probability density function $g(A)$ as well as M_i, $p(M_i)$ and $g(A|M_i)$ for $i = h, s$.
2. The parties choose the contract type which maximises the total surplus and possibly make a down payment to redistribute rents. If the parties choose to sign a contract of type p, the contract is signed.
3. The value of A is revealed to the worker and the value of M to the firm.
4. If the chosen contract is of type e or f, the party that should set the wage offers a wage.
5. If the contract was of type e or f, the party which did not set the wage decides whether to accept or reject and choose its outside option. If the contract was of type p, both parties consider whether to continue the relation in accordance with the contract or choose the outside option so that separation occurs.
6. Production takes place and the worker gets his pay.

To be able to determine which contract type the parties will choose, given the probability density functions for A and M, it is necessary to examine which one maximises the total surplus.[2]

[2]The characteristics of a certain labour market may be rather stable over time. Then one of the wage setting processes will mostly be most efficient and this will tend to standardise the way in which the labour market functions. This is one explanation as to why one rarely sees workers or firms considering which contract type to choose.

I first examine the contract of type p. When the wage w_p is set, the parties are uninformed of what the outcomes of A and M will be. This may imply a risk for the pay being outside the interval $[A, M]$, which leads to an inefficient separation. Recall that for this contract type there are two reasons to separation. If $M < w_p$, pay exceeds productivity and the firm dismisses the worker. If $A > w_p$, the value of the outside option exceeds the pay and the worker chooses the outside option. It is also possible that both apply, but then $A > M$ must apply and separation has to be efficient. Inefficient retentions are never possible. As Hall and Lazear pointed out, the cost of the policy with no renegotiations will be inefficient separations. Allowing for down payments to redistribute rents, the contract has to maximise the expected surplus with respect to the predetermined wage w_p. When both parties are risk neutral the maximisation problem becomes

$$\max_{w_p} S_p = \max_{w_p} \sum_{i=h,s} p(M_i)\Big((1 - X(M_i, w_p))E(A|M_i) + \\ X(M_i, w_p)\Big(p(A < w_p|M_i)M_i + \int_{w_p}^{\infty} Ag(A|M_i)dA\Big)\Big), \tag{3}$$

in which X, as above, is a dummy function which takes the value 1 when the first argument is larger than or equal to the second and the value 0 otherwise. The first term in (3) is the contribution to the surplus when no separation occurs. The second term is the contribution to surplus when the value of the worker's outside option is so high that he exits although the firm would have liked him to continue. The last term represents the outcomes in which the firm dismisses irrespective of whether he is willing to continue or not. The separation outcomes include outcomes with inefficient separations since $M > A$ can hold. Examination of (3) yields the following proposition.

Proposition 1 *For contracts of type p with a probability of separation lower than 1 the surplus maximising wage rate is either $w_p = M_s$ or $w_p = M_h$.*

Proof Setting the wage $w_p = M_h$ or $w_p = M_s$ are obvious choices since these wage levels yield the highest wage possible and, in consequence, the lowest probability of the worker exiting for a given probability of the employee being dismissed. For any solution different from $w_p = M_h$ or $w_p = M_s$ expression (3) is continuous in w_p and the surplus maximising wage should satisfy the first order condition

$$S_p'(w_p) = \sum_{i=h,s} p(M_i)X(M_i, w_p)g(w_p|M_i)(M_i - w_p) = 0, \tag{4}$$

which implies that if $g(A) > 0$ for $A \in [M_s, M_h]$ for such an optimum to exist the positive effect in high-productivity outcomes $(i = h)$ on the margin has to be as big as the negative effect of a higher probabilities for losses in the outcomes with a low productivity $(i = s)$. However, $(M_s - w_p)$ can never be negative when $X = 1$, because the firm would not keep the worker when that implies losses. In consequence the derivative is always positive in the relevant interval and the only possible solutions are $w_p = M_h$ and $w_p = M_s$.

Q.E.D.

Because of the possibility to make down payments, the surplus maximising solution also maximises the profit. The surpluses are given by

$$S_p(M_s) = \sum_{i=h,s} p(M_i) \left(p(A < M_s|M_i)M_i + \int_{M_s}^{\infty} Ag(A|M_i)dA \right) \tag{5}$$

and

$$S_p(M_h) = p(M_h) \left(p(A < M_h|M_h)M_h + \int_{M_h}^{\infty} Ag(A|M_h)dA \right) + p(M_s)E(A|M_s). \tag{6}$$

The latter solution implies that the firm dismisses the employee in the outcome with low productivity, because $M_s < w_p = M_h$. That the parties will set the wage which gives a higher expected surplus implies that the contract will specify the wage M_h, if and only if $S_p(M_h) > S_p(M_s)$ and

$$p(M_h) \left(p(A < M_h|M_h)M_h + \int_{M_h}^{\infty} Ag(A|M_h)dA \right) + p(M_s)E(A|M_s) - \sum_{i=h,s} p(M_i) \left(p(A < M_s|M_i)M_i + \int_{M_s}^{\infty} Ag(A|M_i)dA \right) > 0. \tag{7}$$

Reorganisation yields the result that the condition for $w_p = M_h$ to yield higher surplus is

$$p(M_h) \int_{M_s}^{M_h} (M_h - A)g(A|M_h)dA > p(M_s) \int_{-\infty}^{M_s} (M_s - A)g(A|M_s)dA. \tag{8}$$

The left side is the positive effect of setting M_h associated with the reduction in the probability for the worker choosing the outside option $(p(A < M_h|M_h) < p(A < M_s|M_s))$. The right hand side is the negative effect resulting from the fact that the firm becomes more likely to dismiss the worker although $A > M$. It is more likely that $\hat{w}_p = M_h$ when a high productivity outcome $M = M_h$ is more likely and when A is more likely to be in the interval $[M_s, M_h]$.

The surpluses in (5) and (6) can also be compared to the expression for the expected surplus $E(\hat{S}^*)$ in (2). The examination shows that with a predetermined wage there will tend to be larger losses due to inefficient separations, the larger the overlap in the outcomes of A and M are. This reflects the fact that by agreeing on the wage in advance, the parties forego the opportunity to use some information on the realisation of A and M when setting the wage.

The alternative is that either party sets the wage when the parties have learnt the value of M and A (contracts of type e and f). However, for such contracts to be more efficient than the contract agreed on in advance, the positive effect of more information has to compensate for the negative effect of an increased number of inefficient separations, because of the exercise of monopoly power by the wage setting party. If the firm sets the wage w_f to maximise profit when it knows M and the distribution of A, it sets w_f below M to get some of the surplus. The worker chooses the outside option if $A > w_f$. The firm foregoes some of the potential surplus through inefficient separations when A is not much smaller than M to get a larger share of the surplus when the outside option of the employee is worse. The gain and the cost should balance each other on the margin. Then the following proposition applies

Proposition 2 *In a contract of type f with a positive probability of retention the wage has to satisfy $(M - w_f)g(w_f|M) = p(w_f < A)$.*

Proof The firm maximises the profit given the value of M:

$$\max_{w_f} \Pi_f = \max_{w_f} \int_{-\infty}^{w_f} (M - w_f) g(A|M) dA. \tag{9}$$

The first order condition is

$$\frac{\partial \Pi_f}{\partial w_f} = (M - w_f) g(w_f|M) - p(w_f < A) = 0. \tag{10}$$

In consequence, Proposition 2 has to hold.

Q.E.D.

The positive term in (10) is the marginal gain from raising the probability of retention. The negative term is the marginal cost associated with the wage rise because the firm has to pay more also in the outcomes in which the worker would have stayed without the wage raise. The proposition implicitly determines w_f as a function of M given the distribution of A. Obviously $w_f(M_s) < w_f(M_h)$ applies. The expected surplus when choosing the contract type in which the firm sets the wage then becomes

$$E(\hat{S}_f) = \sum_{i=h,s} p(M_i) \left(\int_{-\infty}^{\hat{w}_f(M_i)} M_i g(A|M_i) dA + \int_{\hat{w}_f(M_i)}^{\infty} A g(A|M_i) dA \right). \tag{11}$$

This is a smaller surplus than the surplus in the first best solution. The source of the inefficiency is that, in order to maximise profits, the firm sets w_f so low that there is a positive probability for inefficient exits. These occur if $w_f < A < M_i$. There will be larger surplus losses when there is a high probability for outcomes in which A takes values much lower than M, because then the firm allows the marginal loss due to inefficient separations $(M - w_f)g(w_f)$ to be rather high to take a larger share of the huge surplus in the outcomes with a low value of the outside option. However, variation in A that is associated with variation in the value of M does not cause inefficiencies.

In the third contract type the worker sets the wage to maximise his expected payoff. Let $U(w_e)$ denote the worker's payoff when setting the wage w_e. Then the following proposition applies:

Proposition 3 *In a contract of type e, the worker will set $w_e = M_h$ if $p(M_h|A)(M_h - A) > M_s - A$, and $w_e = M_s$ otherwise.*

Proof The worker maximises his payoff given the value of A and the information on the outcome for M that he infers from it:

$$\max U(w_e) = \max_{w_e} \sum_{i=h,s} p(M_i|A)(w_e X(M_i, w_e) + A(1 - X(M_i, w_e))), \tag{12}$$

in which $p(M_i|A)$ is the probability for the outcome M_i given A and X takes the value one if $M_i \leq w_e$ so that the firm does not dismiss the worker. Thus, the worker only has to choose whether to set $w_e = M_h$ or $w_e = M_s$, because either of them gives the highest wage for a given positive probability of retention. The condition for the worker choosing to set the higher wage is $U(M_h) > U(M_s)$ or

$$p(M_h|A)(M_h - A) > M_s - A. \quad (13)$$

In consequence, Proposition 3 has to hold.

Q.E.D.

Obviously the worker will be more likely to choose the higher wage, the higher A, M_h and $p(M_h|A)$ are and the lower M_s is. This implies that the employee offers a low wage $w_e = M_s$ if he observes an A less than or equal to some critical value $A^* < M_s$ such that

$$p(M_h|A^*) = \frac{M_s - A^*}{M_h - A^*}. \quad (14)$$

The critical value of A thus depends on the conditional probability density function for A. When there is strong correlation between A and M, there is some interval for A in which $p(M_h|A)$ changes fast with A. If the correlation is weak, the probability changes only a little resulting in that it might be optimal for the employee to make the same offer for all values of A. If the employee sets the wage, the expected surplus is

$$E(\hat{S}_e) = \int_{-\infty}^{A^*} (M_s p(M_s|A) + M_h p(M_h|A))g(A)dA + \int_{A^*}^{\infty} (Ap(M_s|A) + M_h p(M_h|A))g(A)dA. \quad (15)$$

There is a risk for inefficient dismissals, because of the worker setting $w_e = M_h$ when $A \in]A^*, M_s]$ and $M = M_s$. This means that $E(\hat{S}_e)$ in general is smaller than the surplus in the first best solution. Unless A is always so small that M_s is mostly optimal, the surplus will be closer the first best surplus when there is more correlation between A and M, because then the worker's observation of A is more informative concerning M and this reduces the interval of A for which the worker prefers sets $w = M_h$, although M_s is a likely outcome. Moreover, it will be more attractive relative to the firm setting the wage, when the variation in A for values lower than M_h is large relative to the variation in M, because the worker is unwilling to risk large losses for the chance of small gains at the cost of the firm. However, it should be noted that the assumption of a dichotomic distribution of M favours the contracts of type e, because it reduces the variation in M; if the employee sets $w_e = M_s$ he can be certain that the firm accepts.

Which contract type maximises the surplus will depend on $p(M_i), i = h, s$ and the conditional probability density function $g(A|M)$. An examination of (11), (15), (6) and (5), as well as the conditions which implicitly determine the wages in the contracts, yields a number of conclusions. These are summarised in the propositions below.

Proposition 4 *If the distributions of M and A do not overlap, contracts of type p attain first best efficiency. Moreover, $E(\hat{S}_p) > E(\hat{S}_f)$ and $E(\hat{S}_p) \geq E(\hat{S}_e)$.*

Proof If $M > A$ for all outcomes of A and M, there is some value of w_p such that $A < w_p < M$ meaning that there are no (inefficient) separations. The surplus is equal to the surplus in first best contracts. For contracts of type f the firm lowers the pay until the marginal gain of extracting more profit from the firm is equal to the marginal loss from the increase in inefficient exits. Because

$g(A)$ is continuous this means that the probability for an inefficient separation always exists. For contracts of type e, there is a risk for inefficient separations, if setting the higher wage $w_e = M_h$ maximises the employee's payoff. When $M_s - A$ is small relative to the expected gain of of setting $w = M_h$, the employee will set the higher wage M_h which leads to a positive probability for inefficient separations. In consequence, Proposition 4 has to hold.

Q.E.D.

That contracts of type e also sometimes attain first best surpluses is only a consequence of that M is discretely distributed. Proposition 4 corresponds to the main conclusion of Hall and Lazear; that fixed wage contracts will be more efficient. However, there are circumstances under which it will be more efficient that the firm or the worker sets the wage.

Proposition 5 *As the variation in M and A grows larger and the distributions overlap, contracts of type p become less efficient relative to contracts of types e and f.*

Proof When the variation grows large so that the distributions overlap it becomes less and less likely that the predetermined wage is in the interval $[A, M]$, although a separation is inefficient. When the one who sets the wage knows either A or M there is no such increase in the likelihood of separation, because a lower probability density leads to a higher cost associated with a given change in the probability of separation. In addition, knowing A and M becomes more useful, because the gain in expected surplus of knowing one of them increases. In consequence, Proposition 5 has to hold.

Proposition 6 *A larger variation in M tends to make contracts of type f more attractive relative to contracts of type e. A larger variation in A tends to make contracts of type e more attractive relative to contracts of type f.*

Proof Knowing the variable which varies more gives more information. The one who sets the wage has more to gain on risking an inefficient separation when the variation in the other variable is larger. For most changes in the probability distribution this makes the expected loss due to inefficient separations larger the larger the variation in the unknown variable. Ceteris paribus, variation in the outcome of the known variable does not affect the separation risk, because the outcome is known when the wage is set. In consequence, Proposition 6 has to hold.

Q.E.D.

The impreciseness of the above proposition reflects the fact that $g(A)$ can take many forms and changes do not always show up as changes in the variance of the value of A. The same holds for the covariance between M and A.

Proposition 7 *When there is overlapping for the distributions of M and A a stronger correlation between M and A tends to make contracts of type p less likely to be more efficient than contracts of types e and f.*

Proof When the correlation between A and M is strong, a high value of A is also likely to mean a high value of M. This means that when A is much lower than the predetermined wage, M is also more likely to be lower than it. Similarly, when M is much higher than w_p, A is also more likely to be higher than it, although $M > A$. This means that given the variation in A and M, there is more likely to be inefficient and costly separations when the correlation between them is strong. The informativeness of them will also increase. However, this does not necessarily hold for all probability distributions. A stronger correlation may also be associated with such a change in the distributions that chancing on extracting more of the surplus becomes more profitable. In consequence, Proposition 7 has to hold.

Q.E.D.

The propositions above describe how the wages should be set to maximise the expected surplus when there are asymmetries in the information but no negotiations. The wage was assumed to be completely fixed when the contract had been offered and accepted. This means that renegotiation costs and costs for acquiring information are avoided, but that there also are inefficient separations. For contracts of type p, this implied that the parties had no possibility of taking into account the outcomes concerning A or M.

However, as Hall and Lazear pointed out, when contracts are agreed on in advance it might be possible to reduce the risk of inefficient separations by indexing the pay of the employees to variables which are correlated with the payoff of the outside option of the employees or the value for the firm of keeping them. Such indexation will be considered in the next section.

When considering the relevance of the results, one should observe that the actors on the labour market are not always free to set the wage to maximise the surplus or their payoff in the period. There are long term employment relationships in which the wage is rigid and there may be collective agreements which determine the wage. The possibilities to change the wage in response to new circumstances may be limited. Knowing this, the firm will also be unwilling to raise the base wage in response to higher M, because it expects that it will be difficult to lower the base wage later. An alternative strategy is to add a flexible term to the base wage, which is given from the previous period or a collective agreement. This might be accomplished if the flexible term is not included in the contracts and it is explicitly stated that the firm can unilaterally determine the conditions for such pay in future periods. Alternatively, one can assume that the firm has consistent preferences on how the flexible term should depend on some variable, so that the firm can follow the same rule over time. By indexing this pay to some measure of M, the firm can adjust pay according to the value of keeping the worker. Profit sharing may be one way to make pay more responsive to the needs of the firm. The following section examines how large share of the profit the employee should get for the firm to maximise profit. The aim of the firm will thus remain the same, but the firm uses profit sharing to attain it.

3 A profit sharing model with exogenous base wage

In the previous section and in Hall and Lazear (1984) it was assumed that the contract applies for one period of fixed length. In reality, individual employment contracts usually last a length of time, which is not specified in advance. During an employment relationship there are small adjustments of the conditions but because of wage rigidities the wage level in one period strongly affects the wage level in the next. This means that when the parties agrees on a wage, it will influence the wage level long afterwards. One implication of this is that the negotiation power of the parties will be reflected in the wage level, because it is not possible to make down payments of the appropriate size. In consequence, the analysis of the previous section has a limited applicability. Although it is informative for contracts of fixed length it does not say much about the possibilities to reduce inefficient separations in lasting employment relations with wage rigidity. To analyse these I assume that the base wage is exogenous and denote it by w. It may be determined by collective agreements or the wage in the previous period. The base wage may thus be different from base wages in the previous section, even though the parties have the same information. To compensate for the reduced possibilities of adjusting the base wage according to circumstances, the firm adds a wage component which is conditioned on the outcome of M in the period. The firm can thus introduce a kind of profit sharing to reduce inefficient separations.

Because the worker cannot separate between the ordinary base wage and a flexible part which is unconditional, it is assumed that there is no positive intercept in the profit sharing scheme. The analysis also disregards the possibility of a scheme in which the worker gets a bonus only if M exceeds some critical limit higher than w but gets a larger share of the production exceeding this critical level. In consequence, this analysis is limited to a simple class of linear profit sharing schemes in which the firm is assumed to be able to credibly commit to paying a share β of the surplus net of the base wage to the employee. This means that there is only one parameter to determine and that the total wage is given by $w_{tot} = w + \beta(M - w)$.

The worker is imperfectly informed about M and, in consequence, his total pay. When it sets β the firm does not reveal any more information about M to the worker than he knows from other sources. The long run consequences for the worker of quitting are assumed to be included in A. For the worker, the value of A relative to the expected pay when staying in the firm is therefore the only criteria for deciding whether to exit or not. As in the previous section the firm's surplus in its outside option is set to zero.[3] The change in the setting also implies changes in the sequence of events that take place.

Assumption 5 *The sequence of events is as follows*
1. A base wage w is given by an earlier contract.
2. The parties learn the probability density function $g(A)$ as well as M_i, $p(M_i)$ and $g(A|M_i)$, for $i = h, s$.
3. The firm sets the value of β.
4. The value of A is revealed to the worker and the value of M to the firm.
5. The worker decides whether to exit or stay in the firm and the firm decides whether or not to dismiss the worker.
6. If the worker decided to stay and retention was also preferred by the firm, production takes place, M is verified, and the worker gets his pay.

The events before the firm setting β are taken as exogenous in this analysis. The expected profit for the wage level w when $\beta = 0$ and the worker exits if $M > A$ is

$$E(\Pi) = \sum_{i=h,s} p(M_i)X(M_i, w)p(A < w|M_i)(M_i - w). \tag{16}$$

Because the decision criteria of the worker is whether $E(w_{tot}|A) = w + \beta(E(M|A) - w) > A$ or not, a higher β reduces the probability of exit. The worker uses his information about A and β

[3]The value of the outside option of the firm can be seen as the expected productivity of a worker minus the costs of finding and educating a new worker. It includes losses in production and potential expected costs due to the separation in coming periods. The relation between M and the gross surplus is assumed to be so simple that there is also a corresponding simple scheme which is contingent on the gross profit which the worker observes. To keep expressions short I analyse the reduced form of the model, in which only $M - w$ denotes the profit net of the base wage.

to determine the total expected pay. The firm has to take into account that the worker estimates M on the basis of A. There is some range for A over which the worker thinks that the expected total pay in the firm is higher than the value of the outside option so the worker prefers to stay. Using the dummy function X, which takes the value 1 if the first argument is at least as large as the second, the profit maximisation problem becomes

$$\max_{\beta} \sum_{i=h,s} p(M_i)X(M_i,w)p(A < E(w_{tot}|A)|M_i)(1-\beta)(M_i - w) =$$

$$\max_{\beta} \sum_{i=h,s} p(M_i)X(M_i,w)\int_{-\infty}^{\infty} X(E(w_{tot}|A),A)(1-\beta)(M_i-w)g(A|M_i)\,dA. \tag{17}$$

Note that $X(M_i,w) = X(M_i,w_{tot})$, because $w > M_i$ applies if and only if $w_{tot} > M_i$. I assume that the expression is differentiable in β and there is an interior solution to the first order condition. To guarantee an interior solution, pay has to be so weakly dependent on M in optimum that there are outcomes for A above which the value of the outside option becomes higher than the expected total pay of the worker. Thus $X(E(w_{tot}|A),A) = 0$ has to apply for some value of A for the optimal choice of β. Assume that A and M are so weakly correlated that there is only one critical value A^*, above which this applies and the worker exits. For the critical value A^* the expected payoff in the outside option has to be equal to the expected pay in the firm. If $w < M_s$ this implies

$$A^* = w + \beta(M_s - w) + \beta p(M_h|A)(M_h - M_s). \tag{18}$$

Then the maximisation problem becomes

$$\max_{\beta} \sum_{i=h,s} p(M_i)X(M_i,w)\int_{-\infty}^{A^*} (1-\beta)(M_i-w)g(A|M_i)\,dA. \tag{19}$$

The critical value A^* obviously has to be increasing in β. The first order condition of the maximisation problem is

$$\frac{\partial E(\Pi)}{\partial \beta} = \sum_{i=h,s} p(M_i)X(M_i,w)(M_i-w)$$
$$\left((1-\beta)g(A^*|M_i)\frac{\partial A^*}{\partial \beta} - \int_{-\infty}^{A^*} g(A|M_i)\,dA\right) = 0. \tag{20}$$

This condition implicitly determines the profit maximising value of β, which is denoted by $\hat{\beta}$. Setting $\beta = 0$ in (20) and noticing that the derivative then should be nonpositive if $\hat{\beta} = 0$ yields the condition

$$\frac{\partial E(\Pi)}{\partial \beta}\Big|_{\beta=0,A^*=w} = \sum_{i=h,s} p(M_i)X(M_i,w)(M_i-w)$$
$$\left(g(w|M_i)\frac{\partial A^*}{\partial \beta}\Big|_{\beta=0,A^*=w} - \int_{-\infty}^{w} g(A|M_i)dA\right) \leq 0, \tag{21}$$

if $\hat{\beta} = 0$. It is obvious that the profit sharing parameter does not matter if $w \geq M_h$. The other two alternatives are that $w \in]M_s, M_h[$ and $w \in [0, M_s]$. In the former case $X(M_s,w) = 0$ and the outcomes in which $M = M_s$ can be disregarded. Then the condition (21) is equivalent to

$$g(w|M_h)\frac{\partial A^*}{\partial \beta}\Big|_{\beta=0,A^*=w} \leq \int_{-\infty}^{w} g(A|M_h)dA = p(A < w|M_h). \tag{22}$$

This condition is more likely to be satisfied when there is a high probability for that A is smaller than w. The condition is also more likely to be satisfied when $g(w|M_h)$, the likelihood for $A = w$, is small. Under these circumstances there are large costs of setting a positive β but small gains from it. However, an examination of (22) also shows that $\hat{\beta} > 0$ is possible and that is more likely when M_h is large and the probability for exit can be strongly reduced by profit sharing. The derivative $\frac{\partial A^*}{\partial \beta}|_{\beta=0,A^*=w}$ depends on the size of the net surplus $M_h - w$.

When w falls below M_s, the condition for $\hat{\beta} = 0$ changes, because $X(M_s, w)$ becomes equal to 1. This means that

$$p(M_s)(M_s - w)\left(g(w|M_s)\frac{\partial A^*}{\partial \beta}\Big|_{\beta=0,A^*=w} - \int_{-\infty}^{w} g(A|M_s)dA\right) \tag{23}$$

is added to $p(M_h)\frac{\partial E(\Pi)}{\partial \beta}|_{\beta=0,A^*=w,M=M_h}$. The expression in (23) is nonpositive if and only if

$$g(w|M_s)\frac{\partial A^*}{\partial \beta}\Big|_{\beta=0,A^*=w} \leq \int_{-\infty}^{w} g(A|M_h)dA = p(A < w|M_s). \tag{24}$$

This condition obviously has to be satisfied under circumstances corresponding to those for (22), that is when $g(w|M_s)$ and $\frac{\partial A^*}{\partial \beta}|_{\beta=0,A^*=w}$ are small enough and the probability for the worker accepting to stay (the right hand side) is large.

For any given w, there are parameters and probability density function which make the marginal effect of β for $\beta = 0$ on the expected profit given M_s or M_h negative or positive. When $p(M_h)$ or $p(M_s)$ is large enough and the condition in (22) or (24) respectively is satisfied, the marginal effect of raising β above zero on profit is negative, $\hat{\beta} = 0$ has to hold. For a large range of values for the parameters $\hat{\beta} > 0$ will be true. The following proposition gives more information:

Proposition 8 *Take M_h and M_s as given. For all values of w $\hat{\beta} = 0$ is possible, if $p(A < w)$ is high enough, although $p(w < A < M) > 0$. If $p(A > w) = 1$ and $p(w < A < M) > 0$, $\hat{\beta} > 0$ must hold.*

Proof Examine the conditions in (21) – (24). When $p(A < w)$ is high enough and $g(A)$ is low enough for $w < A < M$, the negative effect of unnecessary payments in the outcomes in which the employee would have stayed anyway has to outweigh any positive effect of increased probability of retention for all $\beta > 0$ and $\hat{\beta} = 0$ must hold. It is obvious that $p(w < A) = 1$ implies that $\hat{\beta} > 0$ must hold. In consequence, Proposition 8 has to hold.

Q.E.D.

The results above indicate that a higher variation in A may lead to a lower β, if the higher variation means that $p(A \in [w, M])$ declines. Using earlier results it is possible to prove the following proposition.

Proposition 9 *Even if the worker would like to lower w, given $\beta = 0$, to raise his expected payoff by raising the probability of retention, it is possible that the firm, given w, would like to raise the employee's expected pay by setting $\beta > 0$.*

Proof If $p(M_s)$ is high enough, $A > M_s$ is unlikely enough, and $E(M_s - A|M_s > A)$ large enough the worker would like to set $w = M_s$ in accordance with the analysis in the previous section. If the exogenous wage is higher than M_s the worker would thus like to lower it. However, when $w > M_s$ the firm attach no weight to $p(M_s)$ in deciding $\hat{\beta}$. According to proposition 8 it is possible to find probability density functions such that $\hat{\beta} > 0$. In consequence, Proposition 9 has to hold.

Q.E.D.

It is obvious that $\hat{\beta} < 1$ also has to hold, because when $\beta = 1$ there is no profit. However, to further determine how exogenous variables and parameters affect $\hat{\beta}$, one also has to determine how changes in β affect profit. Differentiating the first order derivative in (20) once more yields the second order derivative. Because $\frac{\partial^2 A^*}{\partial \beta^2} = 0$, it can be written

$$\frac{\partial^2 E(\Pi)}{\partial \beta^2} = -\sum_{i=h,s} p(M_i)X(M_i, w)(M_i - w)$$
$$\left(2g(A^*|M_i)\frac{\partial A^*}{\partial \beta} - (1-\beta)g'(A^*|M_i)\left(\frac{\partial A^*}{\partial \beta}\right)^2\right), \tag{25}$$

which is negative, if $g'(A^*|M_i)$ is nonpositive, which is not necessarily satisfied. To ensure that $\hat{\beta}$ is uniquely determined, the following assumption is made

Assumption 6 *The probability density function $g(A|M_i)$ is such that the largest $\beta \in]0, 1[$ satisfying the first order condition in (20) maximises the profit.*

It is obvious that the largest β satisfying the first order condition is a maximum, because the profit has to be larger than for $\beta = 1$, when the worker is given all the net surplus. In consequence, the second order condition also has to be satisfied. The possibility that $g(A)$ has a large positive slope at some point causing multiple solutions to the first order condition is disregarded. These results are needed for the determination of the effects of other variables.

In addition to being influential on whether it is profitable to use profit sharing and set a β larger than zero or not, the probability distribution of the value A of the worker's outside option is also decisive for the size of $\hat{\beta}$. The first order condition in (20) can also in optimum be expressed as:

$$\hat{\beta} = 1 - \frac{\sum_{i=h,s} p(M_i)X(M_i, w)(M_i - w)p(A < A^*|M_i)}{\sum_{i=h,s} p(M_i)X(M_i, w)(M_i - w)g(A^*|M_i)\frac{\partial A^*}{\partial \beta}}. \tag{26}$$

Examination of the expressions for the first order condition in (20) and (26) yields the following proposition:

Proposition 10 *A probability density function $g(A|M_i)$ that leads to a high probability for values of A close to M_i and a low probability for values of A close to w leads to high values of $\hat{\beta}$.*

Proof Examine the first order condition in (20). High values of the probability density function close to M_i mean higher values for the positive term in the first order condition for high values of β. However, the positive term approaches 0 as β approaches 1. Similarly, a low probability for values close to w means that the value of the negative term is close to zero for low values of β and A^*. When β grows larger, the positive effect on A^* means that the negative term grows larger as well. In consequence, a probability density function which leads to a high probability for values of A close to M_i and a low probability for values close to w has to imply that there has to be a large β satisfying the first order condition. According to Assumption 6 this has to be $\hat{\beta}$. In consequence, Proposition 10 has to hold.

Q.E.D.

Although either $\hat{\beta} = 0$ or $\hat{\beta} > 0$ can hold for any value of w, the base wage clearly has an effect on $\hat{\beta}$. Examination of the first order condition leads to the conclusion that there are a number of effects of changes in w. A higher w reduces the expected surplus net of the base wage. However, a higher w may also change the probability for retention and the likelihood for an outcome in which the worker's expected payoff is equal to the value of the outside option. Moreover, it reduces the firm's profit of keeping the worker in a marginal outcome. The effect of the former depends on the probability density function $g(A)$. The latter reduces the gains of raising β but also reduces the costs of doing it.[4] Further examination of the maximisation problem yields the following proposition

Proposition 11 *If* $(1-\hat{\beta})g(A^*|M_s)\frac{\partial A^*}{\partial \beta} - p(A < A^*|M_s) \geq (1-\hat{\beta})g(A^*|M_h)\frac{\partial A^*}{\partial \beta} - p(A < A^*|M_h)$, *the derivative of* $\hat{\beta}$ *with respect to* w *has to be negative.*

Proof Differentiation of the first order condition yields the derivative of $\hat{\beta}$ with respect to w. Using the expression for A^* in (18) it can be written[5]

$$\frac{\partial \hat{\beta}}{\partial w} = -\left.\frac{\frac{\partial^2 E(\Pi)}{\partial \beta \partial w}}{\frac{\partial^2 E(\Pi)}{\partial \beta^2}}\right|_{\beta=\hat{\beta}} =$$

$$\frac{\sum_{i=h,s} p(M_i) X(M_i, w) \begin{pmatrix} p(A < A^*|M_i) - (M_i - w) g(A^*|M_i) \frac{\partial A^*}{\partial w} - \\ (1-\hat{\beta}) \Bigg(g(A^*|M_i) \frac{\partial A^*}{\partial \beta} - (M_i - w) \\ \Big(g(A^*|M_i) \frac{\partial^2 A^*}{\partial \beta \partial w} + g'(A^*|M_i) \frac{\partial A^*}{\partial \beta} \frac{\partial A^*}{\partial w} \Big) \Bigg) \end{pmatrix}}{\sum_{i=h,s} p(M_i) X(M_i, w)(M_i - w) \begin{pmatrix} 2g(A^*|M_i) \frac{\partial A^*}{\partial \beta} - \\ (1-\hat{\beta}) g'(A^*|M_i) \left(\frac{\partial A^*}{\partial \beta}\right)^2 \end{pmatrix}} =$$

[4] Notice that although there is a discontinuity in the probability for retention for $w = M_s$, this does not mean that there is a jump in the implicit function for $\hat{\beta}$, because the weight of the low productivity outcome declines to zero as w approaches M_s.

[5] Long sequences of terms in the numerator and denominator will for readability be written on several rows inside parentheses.

$$\frac{\sum_{i=h,s} p(M_i)X(M_i,w) \left(\begin{array}{l} p(A < A^*|M_i) - (1-\hat{\beta})g(A^*|M_i)\frac{\partial A^*}{\partial \beta} + \\ (1-\hat{\beta})(M_i - w) \\ \left((1-\hat{\beta})g'(A^*|M_i)\frac{\partial A^*}{\partial \beta} - 2g(A^*|M_i)\right) \end{array} \right)}{\sum_{i=h,s} p(M_i)X(M_i,w)(M_i - w)\left(\frac{\partial A^*}{\partial \beta}\right) \left(\begin{array}{l} 2g(A^*|M_i) - \\ (1-\hat{\beta})g'(A^*|M_i)\left(\frac{\partial A^*}{\partial \beta}\right) \end{array} \right)}, \quad (27)$$

in which the derivatives are taken for $\beta = \hat{\beta}$. The second derivative with respect to β is negative around the uniquely determined endogenous $\hat{\beta}$ and, in consequence, the sign of the numerator determines the sign of the derivative.

The factor $(1-\hat{\beta})g'(A^*|M_i)\frac{\partial A^*}{\partial \beta} - 2g(A^*|M_i)$ has to be negative according to the second order condition when summed over the outcomes $i = h, s$. This implies that the contribution of the third term inside the large parenthesis in the numerator has to be negative. The contribution of the first two terms $p(A < A^*|M_i) - (1-\hat{\beta})g(A^*|M_i)\frac{\partial A^*}{\partial \beta}$ depends on the probability density function g and the critical value A^*. Note that this expression is identical to the last factor in the first order condition in (20). What differs is that the outcomes in the first order condition are weighted with the factor $M_i - w$. For the first order condition to hold, the last factor has to nonegative in one outcome and nonpositive in the other. The condition in this proposition means that it has to be positive in the high-productivity outcome. Remove the factor $M_i - w$ from the expression in the first order condition. What remains of the expression has to nonnegative, because removing the factor $M_i - w$ reduces the weight of the high-productivity outcome. Then the contribution of the terms $p(A < A^*|M_i) - (1-\hat{\beta})g(A^*|M_i)\frac{\partial A^*}{\partial \beta}$ in the numerator above has to be nonpositive. In consequence, the numerator has to be negative under the condition in Proposition 11 and it has to hold.

Q.E.D.

The explanation to these results is that a higher base wage in general tends to reduce the marginal gain of raising β, although it also reduces the cost by lowering the surplus net of the base wage. The former effect usually dominates so that the derivative is negative. The direct effects are rather intuitive, since a larger β means that a change in w has a smaller effect on A^* and a higher $p(A < A^*)$ means that higher base wage more strongly reduces the bonuses the firm has to pay. A negative derivative $g'(A^*|M_i)$ means that a marginal increase in the total wage does not greatly increase the potential of increasing the probability for retention by raising the profit sharing parameter.

However, when the surplus net of the base wage is low in the low productivity outcome, the rise in the base wage greatly reduces the importance of the low productivity outcomes for the determination of the optimal level of profit sharing. If it were much more profitable to keep a high level of profit sharing, given that productivity is high, this may change the result so that the derivative $\frac{\partial \hat{\beta}}{\partial w}$ becomes positive, because then the condition regarding the relation between $g(A^*|M_i)$ and $p(w_{tot} < A^*)$ for the outcomes $i = h$ and $i = s$ do not necessarily hold.

If $w > M_s$, the results will be simpler, because then there will be no weighting of the outcomes. The results above are a consequence of the linearity constraint that is set on the profit sharing scheme. If nonlinear schemes were allowed they would often be preferred in the absence of differences in measurement and administrative costs (see Proposition 14). A greater number of productivity outcomes would greatly complicate the issue.

Not only the base wage but also the other variables affect $\hat{\beta}$. Given the distribution of A conditional on M_i, an increase in total surplus in the outcome M_i raises the profitability of keeping the employee at a given wage but it also raises the cost of giving the employees a share β of the surplus in the outcomes when the worker stays. A closer examination gives the following proposition

Proposition 12 *The effects of M_i on $\hat{\beta}$ is more negative if $g(A|M_i)$ is large for $A < A^*$ but small for larger values of A. A negative value of $g'(A^*)$ also means a more negative effect of M_i. Under opposite circumstances the effect of M_i on $\hat{\beta}$ might be positive.*

Proof Presupposing that $w < M_i$, differentiation of the first order condition in (20) with respect to β and M_i yields

$$\frac{\partial \beta}{\partial M_i} = -\frac{\frac{\partial^2 E(\Pi)}{\partial \beta \partial M_i}}{\frac{\partial^2 E(\Pi)}{\partial \beta^2}} = \frac{p(M_i)\begin{pmatrix} -p(A < A^*|M_i) + (1-\hat{\beta})g(A^*|M_i)\frac{\partial A^*}{\partial \beta} - \\ (M_i - w)\Big(g(A^*|M_i)\frac{\partial A^*}{\partial M_i} - \\ (1-\hat{\beta})\left(g'(A^*|M_i)\frac{\partial A^*}{\partial \beta}\frac{\partial A^*}{\partial M_i} + g(A^*|M_i)\frac{\partial A^*}{\partial \beta \partial M_i}\right)\Big) \end{pmatrix}}{\sum_{i=h,s} p(M_i)X(M_i,w)(M_i - w)\begin{pmatrix} 2g(A^*|M_i)\frac{\partial A^*}{\partial \beta} - \\ (1-\beta)g'(A^*|M_i)\left(\frac{\partial A^*}{\partial \beta}\right)^2 \end{pmatrix}}. \quad (28)$$

The sign of this derivative depends on the sign of the numerator. Contrary to the expression in the previous proof only one outcome concerning M directly affects the numerator. Examination of the expression yields the conclusion that if the outcomes of A, given M_i, are concentrated below A^* so that $g(A^*|M_i)$ is high for these values and the conditional probability density for $A \geq A^*$ is small, the numerator is more negative because the negative terms are larger. If $p(A \geq A^*|M_i)$ is high, it is obviously possible that an increase in M_i makes it profitable for the firm to raise β in order to prevent exits in these outcomes as well. A more negative derivative of the conditional probability density function for $A = A^*$ will obviously tend to make the derivative $\frac{\partial \beta}{\partial M_i}$ more negative. In consequence, Proposition 12 has to hold.

Q.E.D.

A rise in M_i may possibly make it profitable to keep employees in outcomes in which it was not profitable before. Because A^* depends on both outcomes for M, a change in M_s or M_h can also change the weight given to the outcomes in the determination of $\hat{\beta}$.

The probability of the productivity outcomes also affects $\hat{\beta}$. The effect on $\hat{\beta}$ of a higher $p(M_h)$ depends on whether a higher β is more profitable in the high-productivity outcome than in the low-productivity outcome. The extent to which A covaries with M also tend to affect $\hat{\beta}$. In any case a shift upwards for $g(A|M_h)$ or $g(A|M_s)$ may also have a negative effect on $\hat{\beta}$. Hall and Lazear argued that a stronger correlation between the two variables would lead to a stronger dependence of pay on the indexation variable. Although, this may often apply, it does not always have to be the case.

Proposition 13 *An increased covariance between A and M shift may reduce $\hat{\beta}$.*

Proof Easily seen from an examination of the first order condition in (20). If $g(A|M_s)$ shifts upwards, it will often lead to both higher $\hat{\beta}$ and lower covariance. However, also if $g(A|M_h)$ shifts so high that the expected profit of employing the employee falls in the high productivity outcome, there may be an increase in the covariance but a reduction in $\hat{\beta}$.

These results are also dependent on the linearity constraint on the profit sharing scheme. It is easily seen that the following proposition applies.

Proposition 14 *In the absence of differences in administration and measurement costs a profit sharing scheme, which make the profit sharing parameter contingent on M, will lead to higher profits, unless both the term for $i = s$ and the term for $i = h$ in the first order condition in (20) are zero for the same $\hat{\beta}$.*

Proof If the terms are not both zero, it is possible to raise the profit by lowering β for either $i = h$ or $i = s$ and raising β in the other. In consequence, Proposition 14 has to hold.

Q.E.D.

This analysis has not examined any measurement costs associated with the profit sharing scheme. Nonlinear profit sharing schemes may have higher administration costs, especially if the number of potential outcomes for M and the number of payment parameters grow large. However, there are also simple schemes such that the worker gets a fixed sum if the productivity exceeds a certain level. In the model this might correspond to paying the worker more than the base wage only in the high productivity outcome. Considering such possibilities would considerably complicate the analysis. Nevertheless, administrative costs may be an important explanation to the design of profit sharing schemes.

Although a number of simplifications have been made, the analysis has clearly shown that a large number of circumstances affects the optimal use of profit sharing. Use of profit sharing programs is likely to be profitable for firms if the fixed wage is so low that the workers are likely to exit and the fixed wage is rigid. The more valuable the workers are, the larger share of the profit the firm is likely to pay them. However, raising the profit sharing parameter not only prevents exits in certain outcomes. It also raises the wage of the employee for outcomes in which he would have stayed anyway. Therefore, the profit maximising β depends in a complicated way on the exact form of the probability distributions for the value of the outside option and the productivity.

4 Discussion

By analysing the problem in more depth in this paper than Hall and Lazear (1984) I have shown that profit sharing schemes can reduce the risk for inefficient separations when there is rigidity in the base wage. I have argued that with continuing contracts down payments cannot be used to attain a certain distribution of the surplus. Rather the parties have to change the wage rate to change the distribution of the surplus. Wage rigidity makes reductions in the base wage difficult and, in consequence, the firm is not willing to raise the base wage, although there would be a temporary advantage of it. This is one justification for the assumption of a exogenous base wage. However, as Teulings and Hartog (1998) point out, an additional argument as to why the base wage can be viewed as exogenous is that wage rates or changes in them are often determined by industry- or even economy-wide agreements on which a firm and its workers have negligible influence. Under such circumstances, profit sharing may be an important way in which the firm can accomplish more flexibility in the pay.

Although the model is very simple, it gives insights to the difficulties of designing a profit maximising profit sharing scheme. It was concluded that a linear scheme with one parameter rarely maximises profit. How the share of the profit should depend on the profit level, depends on how the value of the worker's outside option tends to be associated with the profit of the firm. The different needs of the firms are one explanation to the wide range of profit sharing schemes that exist.

When considering the empirical implications, the definition that M is the surplus net of the surplus in the firm's outside option should be remembered. In the analysis the outside option was set to zero so that M was the surplus net of the firm's outside option. In reality, M is likely to differ from the profit observed. In particular, if the firm's outside option is positively associated with productivity, M will increase less than profit, and because profit is the measure used by the firms, this will be observed as the employees getting a smaller share on the margin of the profit. At what gross profit level the employees start to get a share of the profit depends on at what level they start to contribute more to the profit than what employing a new worker would do.

Profit sharing is not the only way in which pay can depend on the productivity of the worker. If the firm uses piece rates, the pay of the employee will also be correlated with his productivity in the job. However, the pay of the employee will usually not be perfectly correlated with productivity, because of measurement costs. The measurement costs can mean that the firm observes only a part of the employee's output, because it would be too costly to measure all kinds of output. Nonlinearities in the production function, which for example are associated with teamwork, effectively prevents the firm from calculating the value of the output of an individual. Measurement costs can also otherwise completely prevent the use of piece rates by making the measurement costs outweigh the gains (Lazear 1986). It is also possible to use other measures of the collective productivity of the employees than profit. These might be helpful for the firm to get the employees' pay to covary even more with their productivity in the firm.[6]

Thus, correlation between the pay of the employee and the firm's surplus can be achieved in many ways. However, to offer a credible payment scheme the firm should condition the pay on some variable which is observable to the workers too, because the firm cannot credibly commit to a certain policy if deviations from it are unobservable to the employees. It is enough if the value of the variable is observable afterwards, if this is accompanied by a possibility to punish the firm for defecting from the contract, for example by going to court. To accomplish this it is often necessary that the variable is observable to third parties as well. However, reputational concerns may help the firm commit to following a policy, although the employees and third parties observe the outcomes imperfectly and there is no other punishment than loss of reputation.

The expectations of profit sharing are also likely to affect the level of the base wage set in negotiations. If the firm were free to set the base wage and to give a take-it-or-leave-it-offer which included profit sharing, the profit maximising base wage would often be lower than when there is no profit sharing. The same applies also when the base wage is determined in other ways. However, to determine the effects on the base wage, the bargaining process through which the base wage is determined has to be specified, and one should examine the subgame perfect equilibrium of the wage setting process. The effect of the firm's decision concerning the profit sharing scheme will also depend on at what level in the economy the base wage is set.

It is also important to explain why profit sharing schemes have become so much more common in recent decades. This study has disregarded this issue by ignoring the availability of alternative adjustment mechanisms and of variables to which the pay can be indexed. In many countries the

[6] If both variables that are strongly correlated with M and variables that are strongly correlated with A were available, an even better alternative could be to condition pay on both types of variables.

demand for profit sharing as a way to increase flexibility might have risen as the possibilities for changing real base wages have decreased. A lower inflation, abandonment of devaluations, and the rigidity of wages prevent the cutting of the real base wages in response to a drop in demand. Profit sharing then provides a possibility to raise the pay in response to high demand without loosing the possibility of paying lower wages if the demand falls. This might explain the rising popularity of profit sharing in Finland and other European countries which have abandoned the accomodating monetary policy.

The possibility of the firm to commit to paying a share of the profit to the worker has been taken as given. However, the availability of variables on which collective remunerations can be made contingent may be one important explanation to the increased use of profit sharing schemes. The measurement of firms' profits have become more consistent and more difficult to manipulate. Accounting has also become computerised and this has also increased the availability of measures of collective performance. It has therefore become easier for firms to use profit sharing.

Another possibility is that the environment has become more turbulent with greater differences in productivity. This is likely to raise the needs for firm to, possibly temporary, raise the pay when productivity is high. When one firm starts a profit sharing scheme, others may follow in order to be competitive in the labour market. This seems as a reasonable explanation since technology has become ever more important and shocks related to it greatly influence the productivity of workers in different firms.

5 Conclusion

In fixed-length contracts the surplus can be redistributed by down payments. In contracts for continuing employment relations the parties have to change the wage to achieve the redistribution. One way of doing this is to use profit sharing. The analysis has shown that it might be profit maximising for the firm to introduce profit sharing to raise the workers' pay and reduce the risk for inefficient separations, if the base wage is rigid. However, the analysis has also shown that the profit maximising design of the profit sharing scheme depends on a number of circumstances of which the firm may be poorly informed. Even a more positive covariance between the value of the outside option of the worker and the worker's productivity in the firm does not necessarily raise the optimal level of profit sharing. The firm's cost of acquiring information is therefore an important limitation to its design of profit sharing schemes. Variation in the outside options of the workers and the extent to which their value is correlated with the productivity of the workers in the firm explain the wide variety of profit sharing programs and other collective rewards which exists. The difficulties of designing an optimal system from the firm's point of view explains why profit sharing schemes nevertheless often are fairly simple.

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