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Abstract

This study considers the retail banking duopoly in the production of payment services. Switching costs make the old clients locked-in. Network externalities are exhibited due to high transaction costs in making a transfer to another bank's customer, and due to low marginal costs in processing an intrabank transfer. This makes the discounted marginal profits from the old clientele increase in the number of old clients that favours a larger bank. This can encourage the larger bank to capture the new clients which are free of switching costs, even if price discrimination between old and young clients is not allowed. To obtain this result, requires, however, that a maximal price premium extracted from a payment transfer to another bank's customer is small enough. Thus the ordinary result according to which, it is too big sacrifice for a larger bank to lower also the prices of the old clients in order to capture the new clients, can be reversed.

JEL-code: G21, L11, L12

Key words: retail banking, switching costs, network externality.

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[1] This paper was presented in ESEM 1999 (Santiago de Compostela) and in EARIE 1999 (in Torino).

1. Introduction

The transfer of payments requires debiting and crediting clients' accounts. To be able to handle the payments, the modern bank needs software and also physical capital in the form of buildings, computers and so on. The more electronified the process is, the lower the marginal costs are in producing a payment service. To be able to serve a client a bank has to have an access to an client's bank account. The electronic links between the customers together with the procedures which route payments to right address and do appropriate debiting, crediting and respective accounting, form a payment network. The interconnection of the networks guarantees the access from each customer's account to the payment account of the rival's clients also. When the networks are unconnected there is no electronic link from one bank to another.

In modern banking the payments can be operated fully electronically. When the payment transfer is fully electric the client uses the home computer through, for example, the internet connection. The use of the automatic teller machine in making payments or fund transfers also relies on electronic connections. The merchant respectively has an electric link directly to the bank. If the electric networks are not interconnected a client herself or a merchant must by-pass the missing link from a payer's account to a recipient's account. This requires resorting to more or less manual manoeuvre e.g. desk payment. In any case a client can pay by cash also. As we know a cash payment is still a popular way to pay.

But the clients do not necessarily break even, although the networks are interconnected. It is possible that a client incurs additional transaction costs when paying to some other bank's client or that the interbank transfer is of poorer quality than the intrabank transfer. The interbank transfer may, for example, take more time than the intrabank transfer. Secondly, for the banks the processing of payment transfers which require the interbank settlement may be more costly than processing of intrabank transfers.

In this study we consider the forces which promote concentration in the payment service industry. The clients are divided into old and locked-in clients and into new and uncaptive clients. The main concern is to examine whether the existing market share of old clients induces the bigger bank to capture the new clients also. The price level is considered as well. We consider both the case in which the networks are unconnected and the case in which the networks are joined. In focusing on the competition under unconnection, we consider two separate cases. In the first case price discrimination between new and old clients is not allowed. Whereas, in the second case price discrimination is allowed. Finally we consider the equilibrium when two banks have become compatible under the circumstances in which the intrabank transfer is, however, superior to the interbank transfer in terms of quality.

If the existing market share can function as an impediment to competition and if it favours the larger bank, it thus affects also the terms of interconnection necessary to handle payments in an efficient way. The analysis of this study helps us thus to understand also what is the position of the unconnected small bank or what are the terms of entry. If the small bank suffers from the cancellation of the interconnection contract more than the bigger bank, it is obvious that this can be used as a threat in the negotiations concerning the pricing under interconnection and compensations owing to interconnection.

In the basic framework of this section there are assumed to be two banks: typically one small bank and one large bank. The assumed cost structure of the model does not lead necessarily to imperfect competition. We abstract from possible increasing-returns technology. The banks offer identical payment services. The electronic payment service, which the banks offer, is based on the use of own PCs (through e.g. internet) so that the number of ATMs or the location of the bank branch has no influence.[2] We abstract thus also from all those factors related to the location or the other firm specific attributes which through product differentiation stabilize the market and create a nice equilibrium with many firms.

If the banks are not interconnected, the clients can make a fully electronic payment only to the same bank's clients. Paying to the rival-bank's clients, the customer must resort then to the half-manual procedure which includes positive transaction costs. The network-externality in the case considered depends on the size of the clientele. This externality is wholly internalised only, if the banks join their networks. But also under compatibility we can talk about network externality, if the interbank transfer is of poor quality.

We assume that it is costly for the bank's established client to switch to another bank. There are remarkable transaction costs in closing an account with one bank and opening another with a competitor. The switching costs - which may arise also due to psychology and which may be artificially created to impede competition - are assumed to be so great that it is not profitable for the old clients to switch to another bank. A new customer does not face switching costs. In any other

[2] In many studies in bank competition, the location of the service point makes the products different a' la Hotelling (see. e.g. Matutes and Padilla, EER 1994).

aspect the clients are assumed to be identical. Thus we abstract from the possible cross subsidising which arises when the customers differ from each other in the size of their purchases (see. Tarkka, 1995).

In literature the implications of switching costs are analysed widely. Many authors have recognised that switching costs relax competition and increase the price level in a mature market (see Ausubel (1991), Amstrong, Cowan and Vickers (1994) and Economides (1993)). The banks may, however, use a bulk of those profits obtained from captive clients for the competition about new and uncaptive clients. Klemperer (1987) showed that in the two-period model of Bertrand price competition, even a small positive switching cost give enough shelter to set the second period price of the captured ("old") clients at the monopoly level. In the first period the knowledge of this monopoly pricing makes the competitors to set the prices of new customers below the sort run marginal costs. In this highly competitive situation the discounted profits from each customer remain at zero level. Padilla (1992) considers the two-period model with no price discrimination. He assumes that the clients are differentiated in respect to their awareness of the price offers. Surprisingly, in an equilibrium ex ante identical firms have asymmetric market shares. According to Padilla (1992) overall competition is, however, relaxed due to switching costs.

Farrell and Shapiro (1988) and Beggs and Klemperer (1992) analyze the overlapping-generations duopoly and thus avoid many drawbacks related to two-period models. Farrell and Shapiro (1988), showed that in the case where the clients can not be price discriminated, the bigger firm (in duopoly) is satisfied to serve only captured clients. In this model of two overlapping generations the markets shares would alternate so that the firm with no old clients captures always the new generation. This fat cat phenomenon is present also in the more general setting with an infinite-period market, analysed by Beggs and Klemperer (1992). They showed that when the young generation's switching costs are zero, the old are locked-in, and price discrimination is not allowed, the firm who has more captive clients lets the smaller firm capture the young clients. In the duopoly the market shares would finally converge to the steady-state level which is 50 per cent in the symmetry. In the model of Beggs and Klemperer (1992) the prices in an equilibrium are, however, above the competitive level, not least owing to a positive discount factor.

The above results, according to which, the smaller bank is ready for a fierce competition for new clients, may, however, be reversed, if the bank's current profits are constrained from below. Holmström and Tirole (1997), who considered firms' capital constraints, stress that low profits restrict the capital investments of uninformed investors because of the moral hazard problem. In the banking business especially this link is strong. Low profitability encourages the insiders for deceptive behaviour, that makes the uninformed investors require that the insiders should increase their capital investments. Losses may also be a sign of the ongoing abuse use of capital. Therefore the losses related to aggressive market conquering lead easily to the restraint of capital that makes the bank give up the aggressive strategy and shift back to the conduct which emphasises profitability. The solvency ratios set by the central bank makes this shift in the bank's conduct obligatory in the case in which the bank itself is not yet ready for it. For these reasons, it could reasonable to assume that bank's profits are constrained to be non-negative. This constraint would restrict the smaller bank's possibilities to capture new generations, which in turn would increase the bigger bank's profits.

The recent literature has analysed the impacts of switching costs in regular and potentially competitive industries. Farrel and Shapiro (1988) enlarge the discussion to concern also the effects of scale-economies. According to them, the switching costs in combination with large enough fixed costs could lead to a situation where the incumbent firm excludes the entrant. This result is not very surprising, whereas, it is still unclear to what extent the network externality – in the absence of scale economies – could strengthen the bigger firm's incentives to capture the rest of the market.

We focus on the interplay between the network externalities and the switching costs. This extension is natural, because the switching costs are typical in network industries especially. More specifically, we consider the payment service industry, although our results could be generalised to concern some other network industries as well. The results of this study show that network externality in combination with switching costs leads easily to a highly concentrated market structure. The originally larger bank may extend its market share both in the presence and in the absence of price discrimination. These results may apply also when the networks are joined. Then the larger bank is favoured by the poor quality of interbank transfers and by the low marginal costs associated with processing intrabank transfers.

2. The model

There are only two banks: bank A and bank B. The banks are not necessarily interconnected. The clients of bank A pay price t_A for intrabank transfer. This transfer is fully electronic. If bank A's client makes a payment transfer to bank B's client, she must pay price h_B for it when the networks are not connected. This interbank transfer is set by bank B and it is by nature half-manual. When the banks are interconnected A's client pays h_A for the interbank transfer. The freely determined part of this tariff is set by bank A. The interbank payment leads also to transaction costs for a client, governed by a constant m. Under incompatibility m describes poor quality. Thus a fully electric intrabank transfer is preferred to an interbank transfer. When a client pays to another client in the same bank, the fully electric transfer is available. But in paying to another bank's customer only the payment transfer of poorer quality is available.

The clients are either young or old. We assume that there are T generations of old customers and one generation of new customers. All generations are of equal size. A single client is only a negligible small fraction of the whole generation. Let k denote the number of bank A's old generations and let T-k be the number of B's old generations in current period. A young client turns to old after one period. Like Beggs and Klemeperer (1992) we assume that any old client is expected to leave the clientele with probability 1/(T+1), independently of their past history. In every period the number of new clients emerging the clientele is exactly one unit. A young client chooses the bank without any switching costs. The once-and-for-all switching costs for old agents are so large that the old clients are practically locked-in that makes it possible to set their prices at the monopoly level.

We assume that in their expectations concerning the size of different banks' clientele, the new clients take only into account the observed number of old clients in the beginning of each period. If, however, a new client in her decision making is also assumed to take into account the behaviour of other new clients, the outcome of the game would no longer be predictable. Even if the market shares were the same in the beginning of the game, it could be strictly profitable, for example, for bank A to capture the new clients, if all the new clients share the belief that bank A, instead of bank B, is chosen by new clients. Thus, if we require that expectations should be fulfilled[3], there could easily be two different equilibrium.

We abstract from these difficulties by assuming that the new clients evaluate the probability that A's client will pay to another customer in the same bank being k/T. The same probability concerning bank B is (T-k)/T. Theses probabilities are updated at the beginning of each period. This assumption about the clients' expectations stresses the fact that the single new client in her decision making is not ware of and does not take into account the intentions of other new clients. The banks regard the clients' expectations as given. In their own calculations concerning the capture, the banks

[3] The concept used by Katz and Shapiro (1985) for expectations which are ex post fulfilled. Katz and Shapiro (1985) have pointed out also that network externality is contingent on the expected number of network users.

as strategic players, however, take into accounts all the clients: the new and the old. So if bank A will capture the new generation, bank A calculates that the probability that a single client uses A's fully electric service is rather (k+1)/(T+1) than k/T.

The clients demand bank's payment services insofar as the prices are at the reservation level or below it. The highest possible price for bank A's electronic service is t_A^m and for A's half-manual service h_A^m. For bank B the respective price limits are t_B^m and h_B^m. It is assumed that $t_A^m = t_B^m$ and $h_A^m = h_B^m$. From now on these price limits are denoted by t^m and h^m. It is assumed also that $t^m = h^m + m$.

Following Farrel and Shapiro (1988) we assume that the clients are myopic.[4] This means that the new client chooses the bank whose offer concerning the current period is the best. This simplifying assumption can be defended in the context where the firms do not commit to long term contracts. It turns out that the decisions which the new clients make are ex post rational. So, the myopia assumption does not lead to serious consistency problems. Furthermore, when price discrimination is allowed, the price for old clients is foreseen, wherefore the myopia is then in no contradiction with rationality.

2.1. No price discrimination and no interconnection

We assume now that the bank can not discriminate between the old and young clients.

In this setting the price of the interbank transfer paid by A's clients is set by bank B (and vice versa). A young client chooses bank A, if

$$(1) \qquad \frac{k}{T}t_A + \frac{T-k}{T}(h_B + m) < \frac{T-k}{T}t_B + \frac{k}{T}(h_A + m)$$

where m describes the client's transaction costs related to the use of interbank transfer. If the left hand side of (1), denoted by P_A is the same as the right hand side of (1), denoted by P_B, the clients

[4] Padilla (1992) also assumes that the clients are myopic. Beggs and Klemperer (1992) are critical toward this assumption. The central result of their model is, however, obtained even if the customers were myopic. According to them, it is the fact that the firms discount the future which is sufficient to imply that the desire to exploit old customers outweighs the desire to attract the new ones. The result which states that the market shares of the two symmetric firms converge to equality from any starting is obtained as well, although the customers were myopic.

choose arbitrarily between A and B. Statistically half of the new clients are then expected to choose A. Later in the proof of proposition 1, we suppose that if $P_A \approx P_B$, the clients choose arbitrarily, because they can not see small differences between the current values of t_A and t_B. This supposition also guarantees that in nearly symmetric situation in which T-k < k < T-k+1 or k < T-k < k+1, neither A nor B can get advantage in relation to its rival. Condition (1) is symmetric, because the expectations are static in the short run, and thus the derived equilibrium is unique.

Assume that $t_A = t_B$ and that $h_A = h_B$. Bank B's price exceeds A's price if

$$(2) \qquad -\frac{(2k-T)}{T}t_A + \frac{(2k-T)}{T}(h_A + m) > 0.$$

From the condition (2) we see directly that the customers would choose the larger bank if $t_A < h_A + m$. The network externality thus favours the larger bank, which is assumed to be bank A.

With m = 0, the fully electric service would be a perfect substitute for the half-manual service. Because m > 0, the fully electric service is, however, preferred when $t_A = h_A$.

Let c_t be constant marginal cost of producing a fully electric service. The respective cost for a half-manual service is c_h. It is well known that marginal costs in producing a fully electric service are lower than in producing a half-electric (desk) service. Therefore we assume that $c_t < c_h$. We assume that $h^m \geq c_h$ and show that in the special case where $h^m - c_h = 0$, it is the bigger bank which captures the new clients. Furthermore, we show that when d_h ($\equiv h^m - c_h$) increases the "fat cat" effect may become dominant so that the small bank captures the new clients, even if $d_h < t^m - c_t$. We assume that in any case $t^m - c_t > d_h \geq 0$.

In this highly symmetric game two banks can differ from each other only in respect to the size of their old clientele. Denote by $V_{A,t}(k)$ bank A's net present value in period t when it has k old generations. Then bank B's net present value in period t must be $V_{B,t}(T\text{-}k)$. It is assumed that k > T-k, so that A is originally larger than B in terms of the size of the clientele. If bank A captures a new generation in period t but releases during periods t +1, ..., i, A has in period i $(k+1)(1 - 1/(T+1))^i$ generations. But if bank A releases in periods t, ..., i, the number of its old generations in i^{th} period is $k(1 - 1/(T+1))^i$. In the former case B has in i^{th} period $(T\text{-}k+1)(1\text{-}1/(T+1))^i$ generations and in the latter case $(T\text{-} k)(1 - 1/(T+1))^i$ generations.

In the considered context, it is in the banks' interests to set the price of the half-manual service, concerning all clients, at the monopoly level which is h^m. Because the switching costs are high enough so that the old clients are locked-in, and because price discrimination is allowed, all the prices concerning the old customers can be set at the monopoly level. The banks compete with the price for fully electronic services, concerning new customers.

New clients choose bank A, if condition (1) is valid. Then bank A calculates that it captures a new generation and its current profits are

(3) $$\Pi_A = \frac{(k+1)^2}{T+1}(t_A - c_t) + \frac{(k+1)(T-k)}{T+1}(h_A - c_h),$$

where $h_A = h^m$. If, on the other hand, it is bank B who captures the new generation so that (1) is invalid, A's profits are at the most

(4) $$R_A = \frac{k^2}{T+1}(t_A^m - c_t) + \frac{k(T-k+1)}{T+1}(h^m - c_h),$$

In (4) A sets t_A on the monopoly level. In the considered model the bigger banks benefits from the bigger clientele for two reasons. Firstly, its clients suffer from transaction costs relatively less than the rival's clients that lets the bigger bank set the price for fully electronic service at the higher level than its rival. Secondly, the bigger bank produces fully electronic services relatively more than its rival, which favours the bigger bank, because $c_t < c_h$.

In the dynamic framework, also the long terms effect of the decisions should be taken into account. The banks maximise their performance in terms of net present values. Define $X_{A,t}(k)$ to be a strategy in which bank A lets bank B to capture a new generation in current period. Bank A's net present value is then

(5) $$X_{A,t}(k) = R_{A,t}(k) + \delta V_{A,t+1}(k(1 - 1/(T+1))),$$

where R, which denotes the current period profits from old locked-in clients, is defined in equation (4). Respectively, banks B's net present value would then be

(6) $$W_{B,t}(T-k) = \Pi_{B,t}(T-k) + \delta V_{B,t+1}((T-k+1)(1 - 1/(T+1))),$$

where $\Pi_{B,t}(T-k)$ describes B's current period profits from old and young. It has an equation

(7) $$\Pi_{B,t}(t-k)=\frac{(T-k+1)^2}{T+1}(t_B-c_t)+\frac{k(T-k+1)}{T+1}(h_B-c_h),$$

where $h_B = h^m$. Let $W_{A,t}(k)$, on the other hand, denote a strategy in which A captures the new clients in the current period. This strategy has then an expression

(8) $$W_{A,t}(k) = \Pi_{A,t}(k) + \delta V_{A,t+1}((k+1)(1 - 1/(T+1))),$$

where $\Pi_{A,t}(k)$ represent current profits defined in an equation (3). The corresponding net present value for bank B is then

(9) $$X_{B,t}(T-k) = R_{B,t}(T-k) + \delta V_{B,t+1}((T-k)(1 - 1/(T+1))).$$

In (9) the current profits from old clients has an expression

(10) $$R_{B,t}(T-k)=\frac{(T-k)^2}{T+1}(t_B^m-c_t)+\frac{(k+1)(T-k)}{T+1}(h^m-c_h).$$

Define $t_{A,t+h}^{o}$ to be A's price offer in period t+h concerning new clients, and respectively, $t_{B,t+h}^{o}$ B's price offer concerning period t+h as well. The price offer is credible, if the bank can commit to it without losses. For example, bank A can by its price strategy affect bank B's prices insofar as it can stick to its offer in the case A's offer beats B's offer and becomes thus effective. So long as bank A captures new clients, B's profits do not, necessarily, depend on price offer $t_{A,t}^{o}$. If B foresees that it will not be profitable to capture new clients in the future, B is content with taking the full advantage of its old clients by charging for them monopoly price t^m.

It must be noticed that k > T-k. We try to find out, does there exists such a credible price offer $t_{A,t}^{o}$ which makes the client to choose bank A in period t subject to condition $W_{A,t}(k) - X_{A,t}(k) > 0$ and $W_{B,t}(T-k) - X_{B,t}(T-k) < 0$. In other words, it is required that under current prices $t_{A,t}$ and $t_{B,t}(t_{A,t})$, clients choose bank A, to whom the capture is profitable, while the capture would be unprofitable for B. The problem in this setting are the prices offers in the future. Before we go to proposition 1 and its proof, some notation is introduced, in order to express the present net values also as functions of current and future prices.

Define $\mathbf{t_A} = t_{A,t+1}, t_{A,t+1}, t_{A,t+2,...}$ to be the vector of A's future prices when the number of its old generations is k(1- 1/(T+1)) in period t+1. Whereas, $\mathbf{t_A}^* = t_{A,t+1}^*, t_{A,t+2}^* t_{A,t+3}^*{}_{,...,}$ denotes then future prices when the number of A's old generations is (k+1)(1- 1/(T+1)) in period t+1. Respectively, B's prices are then $\mathbf{t_B} = t_{B,t+1}, t_{B,t+2}, t_{B,t+3,...}$ when the number of B's old generations is (T-k)(1-1/(T+1)) in period t+1, and $\mathbf{t_B}^* = t_{B,t+1}^*, t_{B,t+2}^*, t_{B,t+3}^*, ..$, when B has (T-k+1)(1- 1/(T+1)) old generations in period t+1.

When a bank releases from now on, and sets the future prices of old clients at the maximum level, the discounted profits from period t+1 onwards are

$$\delta D_{A,t+1}((k+1)(1 - 1/(T+1)), \mathbf{t_A^m}) = \sum_{i=1}^{\infty} \delta^i R_{A,t+1}((k+1)(1 - 1/(T+1))^i, t_A^m)$$

$$\delta D_{A,t+1}(k(1 - 1/T), \mathbf{t_A^m}) = \sum_{i=1}^{\infty} \delta^i R_{A,t+1}(k(1 - 1/(T+1))^i, t_A^m)$$

(11) $$\delta D_{B,t+1}(T-k+1)(1- 1/(T+1)), \mathbf{t_B^m}) = \sum_{i=1}^{\infty} \delta^i R_{B,t+1}((T-k+1)(1 - 1/(T+1))^i, t_B^m)$$

$$\delta D_{B,t+1}((T-k)(1 - 1/(T+1)), \mathbf{t_B^m}) = \sum_{i=1}^{\infty} \delta^i R_{B,t+1}((T-k)(1 - 1/(T+1))^i, t_B^m)$$

Denote $D_{A,t+1}^{k+1} \equiv D_{A,t+1}((k+1)(1 - 1/(T+1)), \mathbf{t_A^m})$, $D_{A,t+1}^{k} \equiv D_{A,t+1}(k(1 - 1/T), \mathbf{t_A^m})$,

$D_{B,t+1}^{T-k+1} \equiv D_{B,t+1}((T-k+1)(1 - 1/(T+1)), \mathbf{t_B^m})$ and $D_{B,t+1}^{T-k} \equiv D_{B,t+1}((T-k)(1 - 1/(T+1)), \mathbf{t_B^m})$.

Let $t_{A,t+h}^I$ denote such $t_{A,t+h}$ under which A is indifferent between the capture and the release in period t+h when the number of A's old generations is k(1- 1/(T+1)) in period t+1 and when $V_{A,t+1}$ is restricted to be $D_{A,t+1}$. Then $\mathbf{t_A^I} = t_{A,t+1}^I, t_{A,t+2}^I, t_{A,t+3}^I$, .. is the respective price vector. When the number of A's old generations is (k+1)(1- 1/(T+1)) in period t+1 we use notation $\mathbf{t_A^{I*}} = t_{A,t+1}^{I*}$, $t_{A,t+2}^{I*}$, $t_{A,t+3}^{I*}$, ... for the price vector which implies the indifference. The corresponding vectors for bank B are $\mathbf{t_B^I}$ and $\mathbf{t_B^{I*}}$. By these definitions

$V_{A,t+1}((k+1)(1 - 1/(T+1)), \mathbf{t_A^{I*}}) = D_{A,t+1}^{k+1}$, $V_{A,t+1}(k(1 - 1/(T+1)), \mathbf{t_A^I}) = D_{A,t+1}^{k}$,

$V_{B,t+1}((T-k)(1- 1/(T+1)), \mathbf{t_B^I}) = D_{B,t+1}^{T-k}$ and $V_{B,t+1}((T-k+1)(1- 1/(T+1)), \mathbf{t_B^{I*}}) = D_{B,t+1}^{T-k+1}$.

Proposition 1 states the main result of this study. We assume that $h^m - c_h$, is very small in relation to price premium $t^m - c_t$. For simplicity it is assumed that $h^m = c_h$.

Proposition 1. If $h^m = c_h$, then it is the bigger bank who captures the new clients, even if price discrimination is not allowed.

The exact **proof** is given in Appendix. In the proof it is assumed that that both bank A and bank B make the price offers concerning future also and that they both have made price offers which make the rival indifferent between the capture and the release in the future. In other words, $t_{B,t+h}(t_{A,t+h}^{o}) = t_{B,t+h}^{I}$ and $t_{A,t+h}(t_{B,t+h}^{o}) = t_{A,t+h}^{I}$ when $h \geq 1$. If under these assumptions A would capture in the current period, and if it turns out that it will capture also from then on insofar as it is bigger than B, its price offers concerning the future are credible. In fact, bank A's capability to capture in the current period proves that it can also capture in the future and thus set $t_{A,t+h}^{o}$ so that $t_{B,t+h}(t_{A,t+h}^{o}) = t_{B,t+h}^{I}$ or slightly below it. It is shown also that although bank B has made A indifferent in the future, bank B does not capture in current period. This refers to the fact that bank B can capture neither in the future periods. Thus B's price offer concerning the future is not credible that means that for the credible offers $t_{A,t+h}(t_{B,t+h}^{o}) \geq t_{A,t+h}^{I}$. When A captures, $t_{A,t+h} = t_{A,t+h}^{o}$ so that $t_{B,t+h}(t_{A,t+h}) = t_{B,t+h}^{I}$. A makes then B indifferent. Denote the vector $\mathbf{t_A}^{*}(\mathbf{t_B}^{\mathbf{I+}}) - \mathbf{t_A}^{\mathbf{I}*}$ (with elements $t_{A,t+h}^{*}(t_{B,t+h}^{I+}) - t_{A,t+h}^{I*}$) by $\Delta\mathbf{t_A}^{*}$ where $t_{B,t+h}^{I+}$ describes B's indifference price-level when the number of its old generations is T-k-1 in the current period. Then $\Delta\mathbf{t_A} \equiv \mathbf{t_A}(\mathbf{t_B}^{\mathbf{I}}) - \mathbf{t_A}^{\mathbf{I}}$ (with elements $t_{A,t+h}(t_{B,t+h}^{I}) - t_{A,t+h}^{I}$). Insofar as the changes $\Delta\mathbf{t_A}^{*}$ and $\Delta\mathbf{t_A}$ lead to

$$(12) \qquad V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A}^*) - V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A}) >$$

$$V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A}^{\mathbf{I}*}) - V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A}^{\mathbf{I}}),$$

the original result, according to which, bank A captures in the current period, is only confirmed. The higher B's credible price offers in the future are, the more profitable it is for bank A to capture the current new generation. At the first stage of the proof we show that when $t_{A,t+h}(t_{B,t+h}^{o}) = t_{A,t+h}^{I}$ and $t_{A,t+h}^{*}(t_{B,t+h}^{o}) = t_{A,t+h}^{I*}$ (when $h \geq 1$) there exists such an price offer $t_{A,t}^{o}$ and $t_{B,t+h}(t_{A,t+h}^{o}) = t_{B,t+h}^{I}$ which make the client to choose bank A in period t subject to condition $W_{A,t}(k) - X_{A,t}(k) > 0$ and $W_{B,t}(T-k) - X_{B,t}(T-k) < 0$.

At the second stage of the proof we show then that (12) is fulfilled. In the considered setting it is sufficient to show that $\Delta\mathbf{t_A}^{*} \geq \Delta\mathbf{t_A}$. This guarantees that inequality (12) is valid. At the second stage of the proof it is shown also that so long as A charges so that $t_{B,t+h}(t_{A,t+h}^{o})$ is slightly below $t_{B,t+h}^{I}$, bank B cannot expect its net present value $V_{B,t+1}((T- k)(1 - 1/(T+1)))$ of being greater than $D_{B,t+1}^{T-k+1}$, if B captures the current new clients. This claim is defended by showing that in symmetry or nearly symmetry bank B can expect that its net present values is at the highest $D_{B,t+1}^{T-k}$. Owing to ruinous competition, corresponding to unstable situation in the neighbourhood of symmetry, the net present values can easily drop even below this level.

The proposition 1 states that when the profit making opportunities in half-manual transfers are non-existent, the big bank will capture the new clients although price discrimination is not allowed. The proof of proposition 1 shows that by setting the user charge $t_{A,t}$ consistently so that $t_{B,t}(t_{A,t}) = t_{B,t}^I$, the bank A can be assured that B does not capture new clients. When prices are set non-cooperatively and simultaneously there is, however, no Nash-equilibrium for prices. Suppose that A charges along $t_{B,t}(t_{A,t}) = t_{B,t}^I$ and B sets $t_{B,t} = t^m$. Then A has an interest to raise its price offer. If A, however, raises its offer so that $t_{B,t}(t_{A,t}) > t_{B,t}^I$, it is in B's interests to lower its own price all the way below $t_{B,t}(t_{A,t})$, even if B expects that its net present value of future profits do not exceed the break-even level which is the discounted maximum profits from old clients.

It turns out also that the price level tends to raise above the competitive level.

Corollary 1. When $h^m = c_h$, the price margin concerning current price for new clients is at the most

$$t_{A,t} - c_t = (t^m - c_t)\left[\frac{2k-T}{k} + \frac{(T-k)^3}{k(T-k+1)^2} - \frac{(T-k)[2(T-k)+1]}{k(T-k+1)^2}\frac{\delta(1-\frac{1}{T+1})^2}{1-(1-\frac{1}{T+1})^2\delta}\right]$$

The proof is given in appendix. Corollary 1 shows that $t_{A,t} < t^m$. The higher is discount factor δ, the thinner is the price margin. This result shows that the larger present value of an increment in clientele implied by higher δ, makes the banks fight more fiercely for the new clients. Corollary 1 tells also that the bigger is k, the number of the bigger bank's old clients, the wider is the price margin. Price margin $t_{A,t} - c_t$ is almost always positive. Only if δ is close to one and if k is much bigger than T-k, the current price can be negative.

The case without network externality

Above, it was the network externality which resulted in increasing returns on old clients and in the non-existence of a competitive equilibrium. If the network externality is removed, the industry becomes regular and the larger banks starts to behave like a "fat cat" who is satisfied to its old clients.

Let us consider a model where bank A who has originally k old generations and bank B T-k old generations so that k > T-k. Each bank produces only one service with user charges $t_{A,t}$ and $t_{B,t}$. Bank A's monetary earnings from its old generations are at the most $k(t^m - c_t)$ in the current period. The dynamics of the model is assumed to be the same as previously. Because there are no network externalities, the new clients choose A, if $t_{A,t} < t_{B,t}$. We show that there cannot exist an equilibrium in which it pays for A to capture new generations and in which it is not profitable for B to capture new generations.

Propositions 2. In the absence of network externality, it does not pay for A to capture all the new generations in the future.

The proof is given in Appendix. The proposition excludes the possibility that the larger bank captures and thus confirms the previous results (see Farrel and Shapiro, 1988, and Beggs and Klemperer, 1992) who state that when price discrimination is not allowed it is the smaller firm who captures the new clients. It turns out that this result can be obtained even in the presence of network externality, if the price premium $h^m - c_h$, from producing half manual services, is not too low in relation to premium $t^m - c_t$.

The case: $h^m - c_h > 0$

Denote $d_h \equiv h^m - c_h$. We will show that when the profitability of half-manual transfers increases (described by d_h) the terms for capture may switch so that it is no more profitable for the larger bank to capture new generations, although all the time $d_h < t^m - c_t$.

Above it has been assumed d_h is zero or at least very small in relation to margin $t^m - c_t$. When d_h increases in relation to $t^m - c_t$ much enough proposition 1 is possibly no longer valid. In fact, assuming that $t^m - c_t > d_h > 0$, two additional terms should be added to condition (a5) (in appendix) which tells which one of two banks captures new clients. Those new sum terms are

$$-d_h \frac{2k-T}{(T-k)(k+1)^2(T-k+1)^2}[k(T-k+1)^2 + (T-k)(k+1)^2].$$

and

$$d_h[-\frac{(2k-T)(3k(T-k)+2T+1)}{(T-k)(k+1)^2(T-k+1)^2}\frac{\delta(1-\frac{1}{T+1})^2}{1-(1-\frac{1}{T+1})^2\delta}$$

$$-\frac{(2K-T)(K(T-k)-1))(T+1)}{(T-k)(k+1)^2(T-k+1)^2}\frac{\delta(1-\frac{1}{T+1})}{1-(1-\frac{1}{T+1})\delta}].$$

It is noteworthy that above the absolute value of d_h's negative coefficient is greater than the coefficient of (t^m - c_t) in formula (a5). This would suggest that, given (t^m - c_t), it does not pay any more for A to capture the new clients so long as it is bigger than B.

Corollary 2. In the presence of network externality, it does not pay for A to capture all the new generations in the future, if d_h is close to t^m - c_t when $0 < d_h$ ($< t^m$ - c_t).

Proof. Proof follows the proof of proposition 2. In the case considered A would also make losses, if it captures new clients, given that the both banks cannot earn in the future more than the discounted profits (11). If A, however, would decide to capture new clients, $V_{A,t+1}^{k+1}$- $V_{A,t+1}^{k}$ would become smaller than $D_{A,t+1}^{k+1}$- $D_{A,t+1}^{k}$, because A should lower the offers below $t_{A,t}^{I}$ and because $t_{A,t}(t_{B,t}^{I})$ - $t_{A,t}^{I}$ decreases in k. This only confirms the fact that it is not profitable for A to capture the new clients.

Thus, it is shown that the roles of two banks may switch so that the smaller bank will actually capture new generations when the profits from half-manual service relatively increase.

2.2. Price discrimination and no interconnection

In the banking and in the provision of payment services, the price discrimination could take various forms. It is not necessarily reasonable to lower the prices of payment services for that part of clientele which is assumed to be noncaptive. The youngsters do not use payment services so heavily as captive clients do. Technically the price discounts concerning the youngsters only would be rather difficult to carry out. It may thus be more convenient to grant to the young loans on favourable terms or to attract them with lucrative deposit opportunities. Of course the difficulties to identify the new client from the old, and the possible arbitrage which take advantage from the offered price discounts, set limits to price discrimination. We, however, assume next that it is possible to restrict the price discounts to concern the young and uncaptive clients only. The old and captive clients have then to pay monopoly price for the payment services.

An inequality (1) describes still those terms under which the client chooses bank A, whereas when bank A captures the new clients its current profits are

$$\Pi_A = \frac{(k+1)}{T+1}(t_A - c_t) + \frac{k(k+1)}{T+1}(t_A^m - c_t) + \frac{(k+1)(T-k)}{T+1}(h^m - c_h). \tag{13}$$

Bank B's profits, in the case it captures the new clients, are

$$\Pi_{B,t}(t-k) = \frac{(T-k+1)}{T+1}(t_B - c_t) + \frac{(T-k)(T-k+1)}{T+1}(t_B^m - c_t) + \frac{k(T-k+1)}{T+1}(h^m - c_h). \tag{14}$$

Equations (4) and (10) still describe the profits from the old clients. Also the dynamics can be presented in the same way as in the case without price discrimination. It turns out that

Proposition 3. If $h^m = c_h$, then it is the bigger bank who captures the new clients, when price discrimination is allowed.

The proof is in Appendix. The right hand side of inequality (c1) which describes $t_{B,t}^I$, is

$$-\frac{(T-k)}{(T-k+1)}t_B^m + \frac{2(T-k)+1}{(T-k+1)}c_t - \frac{T+1}{(T-k+1)}\delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}).$$

In the strategy in which A captures the new clients, A sets $t_{A,t}$ so that $t_{B,t}(t_{A,t})$ is slightly below $t_{B,t}^I$. The above expression shows that $t_{B,t}^I$ is even negative. Despite this aggressive pricing, the upperbound for the average current price which is $(1/(k+1))t_{B,t}^I + (k/(k+1))t^m$ is even higher than in the case in which price discrimination is not allowed. This result can be derived using the above formula and the right hand sides of inequalities (a3) and (c1) which express $t_{B,t}^I$ in each case.

3.3. Interconnection and no price discrimination

We consider next the situation where the banks are interconnected.[5] If bank A's client pays to another client in the same bank, the price for the transfer is t_A. If A's client pays instead to bank B's

[5] The behaviour of banks under compatibility is analysed also in Kauko (1998).

client, the interbank links must be used. The price for an interbank transfer can be divided into two parts which are h_A‘ and I_A . Then h_A‘ stands for A's share and I_A for B's share of that price for a compensation to process the transfer. It will turn out that both for bank A's and bank B's clients the expected volume of interbank transfers is (k+1)(T-k)/(T+1) and thus the same. We consider an institutional setting in which A and B have agreed on I_A and I_B, and due to reciprocity have set $I_A = I_B$. In so far as competition authorities had any influence on interbank transfers, the above equality would apparently be a minimum requirement. For the simplicity, we assume that $I_A = c_I$ when c_I is the receiving bank's marginal costs to complete the processing of an interbank transfer. Bank A's profits are then

$$\Pi_A = \frac{(k+1)^2}{T+1}(t_A - c_t) + \frac{(k+1)(T-k)}{T+1}[(h_A' - c_h + c_I) + (I_B - c_I)]$$

when c_h describe marginal costs of the whole interbank transfer. Regarding I_A as given, we can write $h_A = h_A' + I_A$, and because $I_A = I_B$, $h_A = h_A' + I_B$ also. Thus the above equation reduces to equation (3). We can still consider t_A and h_A as A's competition instruments, although it is actually h_A‘ which is A's decision parameter.

We consider a situation in which the transfer which uses interbank networks is of quality inferior to the intrabank transfer. Poor quality means delays in transfers, and worse reliability and safety than in intrabank transfers. The income from the float is assumed to be part of h_A. All the expenses from the unit interbank payment for the client are $h_A + m$ when m describes the influence of poorer quality. We still assume that t_A does not exceed the reservation price t_A^m and h_A the reservation price h^m. Also the relation $t^m = h_A^m + m$, is assumed to be valid.

When the banks are interconnected inequality (1) is replaced by inequality

$$(15) \qquad \frac{k}{T}t_A + \frac{T-k}{T}(h_A + m) < \frac{T-k}{T}t_B + \frac{k}{T}(h_B + m),$$

where $h_A = h_A' + I_A$ and $h_B = h_B' + I_B$. When (15) is in force, the client chooses bank A. Equations (3) and (7) describe still A's and B's current profits when they capture the new clients. It is no longer clear that the banks would compete with prices t_A and t_B. Assume that bank A keeps the left-hand side of (15) constant. Then $\partial t_A/\partial h_A = -(T-k)/k$ and $\partial h_A/\partial t_A = -k/(T-k)$. Keeping the user cost (which is the left hand side of (15)) constant, it follows then from equation (3) that

$$\frac{\partial \Pi_A}{\partial t_A} = \frac{(k+1)(2k+1)}{T+1} \text{ and } \frac{\partial \Pi_A}{\partial h_A} = \frac{(k+1)(T-k)}{k(T+1)}.$$

This shows that $\partial\Pi_A/\partial t_A > \partial\Pi_A/\partial h_A$, so that it pays for A to set $t_A = t_A^m$ and compete with h_A given I_A and I_B. Similarly it is obtained for bank B the conditions

$$\frac{\partial \Pi_B}{\partial t_B} = \frac{(T-k+1)^2}{T+1} - \frac{(T-k)k(T-k+1)}{k(T+1)} \text{ and } \frac{\partial \Pi_B}{\partial h_B} = \frac{k(T-k+1)}{T+1} - \frac{k(T-k+1)^2}{(T-k)(T+1)},$$

which describe how much profits change while keeping the price condition concerning the clients constant. It follows from the equations above that $\partial\Pi_B/\partial t_B > \partial\Pi_B/\partial h_B$. Thus it is also in bank B's interests to set $t_B = t_B^m$ and compete with h_B. We will show that also under characterized interconnection the smaller bank is easily loosing its market share, if the quality of interbank transfers is inferior.

Proposition 4. If $t^m = h^m + m$, $h^m = c_h$ and $t^m < c_t$, then it is the bigger bank who captures the new clients, even if the banks are interconnected and if price discrimination is not allowed.

The **proof** of proposition 4 follows closely the lines of the proof of proposition 1 (see Appendix). The proposition shows that the larger bank's aggressive nature does not necessarily vanish when the banks become compatible. But it is clear that the implications of corollary 2 also are valid now. If under compatibility, the interbank transfer does not differ from the client's view point much from the intrabank transfer, and if there are no remarkable differences in marginal costs, it is possible that compatibility would make the larger bank a fat cat. The concentration of the industry would no longer be such a problem which in the long run depresses competition.

Conclusions

We consider bank competition in payment services in the two different situations. In the first case the networks are not joined. Incompatible networks would, however, complement each other. A payment transfer to the same bank's customer is fully electronic. But a transfer to another bank's client is by nature half-manual. When the customer must resort to half-manual service her costs consists of, in addition to the regular user price, transaction costs which arise when one makes payments in the half-manual way. In fully electric transfers the transaction costs are zero. In the second case the networks are joined. Then an interbank transfer is of quality inferior to an intrabank transfer, or at least, the marginal costs to process an interbank transfer are higher than the respective costs in processing an intrabank transfer.

The banks are assumed to compete with prices. The banks differ from each other only in the numbers of their old clients. The switching costs which the customers face are large enough to deter any movement from one bank to another. Only the new clients who enter the market are free from switching costs.

Let us consider first the results in the first case (when the banks are not interconnected). The probability that the client must pay to another bank's client and thus resort to half-manual procedure is then greater for the smaller bank's customer than for the bigger bank's customer which benefits the clients of the bigger bank. The marginal costs of processing fully electronic transfers are assumed to be lower than the respective costs in processing half-manual transfers. Also this favours the bigger banks on the expense of the smaller rivals.

When the price discrimination is not possible, it could be too big sacrifice for a larger bank to lower also the prices of the old clients below the monopoly level in order to capture the new clients. This ”fat-cat” effect is noticed in related literature (see Farrell and Shapiro (1988), and Beggs and Klemperer (1992)). In the case considered the network externality may improve, however, the bigger bank's clients' position to an extent that in order to capture new clients the bigger bank needs lower the prices only moderately. Network externality makes the discounted marginal profits from the old clientele increasing in the number of old clients. These effects dominate the "fat cat" effect, if a price premium associated with the supply of half-manual services is zero or small enough. If this premium increases close to a (maximum) premium associated with the supply of fully electronic transfers, the "fat cat" effect starts to dominate. This refers that there is a threshold-value for the price margin in question. Below this threshold concentration is observed, whereas above this threshold the market shares become smooth. Typically, in the situation in which the larger bank captures all the new clients, the overall price level remains above competitive level.

It pays for the larger bank to capture the new clients also when price discrimination is allowed, if the premium from half-manual services is small. In that case the current price of the fully electronic service for the new clients is even negative. Despite this, the average price of these services for the whole clientele would be even higher than in the case in which price discrimination is not allowed.

One can state that when competitive pricing is restricted to concern new clients only, the larger bank is even better-off than in the absence of price discrimination.

Finally we consider the case in which the payment networks are joined. Because the quality of interbank transfer is inferior to the quality intrabank transfer, and the marginal costs in processing intrabank transfers are lower than in transfers which use interbank links, the larger bank obtains a competition advantage. If the price premium in interbank transfers is small enough, it is profitable for the larger bank to capture the new clients, whereas, if there this premium is almost of the equal size as the premium associated with intrabank transfers, it pays for the larger bank to let the rival capture the new clients.

For two reasons the interconnection of payment networks, however, could favour competition. Firstly, when the networks are joined, the quality difference between an intrabank transfer and an interbank transfer may be smaller than the respective difference between a fully electronic transfer and a half manual transfer in unconnected networks. Secondly, also the differences between the marginal costs in processing an intrabank transfer and an interbank transfer may be smaller than the respective cost difference associated with the processing of a fully electronic and a half-manual transfer. Thus it is possible that concentration occurs when networks are not linked; and that the market become more fragmentary when networks are connected.

In this study we abstract from all factors which generate product differentiation and as such explain stationarity in the market shares. We focus purely on the interplay between networks externality and switching costs. The world of the model considered does not necessarily correspond to that world in which the banks operate currently. But it more or less pictures the world toward which retail-banking is also developing. The geography loses its former importance and the clients rely increasingly to the PC-access in making payments and fund transfers. Our study shows that the stable equilibrium in which each bank keeps its former market share or in which small banks become bigger, may disturb.

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Appendix

The proof of proposition 1

Stage one of the proof

Formally we consider does there exist such $t_{A,t}$ and $t_{B,t}(t_{A,t})$ which make the new generation to choose bank A so that

(a1) $\Pi_{A,t}(k,t_{A,t}) - R_{A,t}(k,t_A^m) + \delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}) > 0$

and

$\Pi_{B,t}(T-k,t_{B,t}(t_{A,t})) - R_{B,t}(T-k,t_B^m) + \delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}) < 0.$

Taking into account equations (6) - (13), inequalities (a1) can be expressed in the form

(a2) $$t_{A,t} > \frac{k^2}{(k+1)^2} t_A^m + \frac{2k+1}{(k+1)^2} c_t - \frac{T+1}{(k+1)^2} \delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k})$$

and

(a3) $$t_{B,t} < \frac{(T-k)^2}{(T-k+1)^2} t_B^m + \frac{2(T-k)+1}{(T-k+1)^2} c_t - \frac{T+1}{(T-k+1)^2} \delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}).$$

The right hand side of (a2) is $t_{A,t}^I$ which defines the level at which A is indifferent between the capture and the release, given it gets in the future at the most only $D_{A,t+1}^{k+1}$ (when A captures in current period) or $D_{A,t+1}^{k}$(when A releases in current period). In the same way, the right hand side of (a3) is $t_{B,t}^I$ which defines a break-even point for B when B's future prospects promise at the most $D_{B,t+1}^{T-k}$ or $D_{B,t+1}^{T-k+1}$. We assume that also in nearly symmetry when $T-k < k < T-k+1$ or $k < T-k < k+1$, $|t_{A,t}^I - t_{B,t}^I| < \eta$, when η describes a negligible small price difference which the client does not take into account.

According to condition (1), a young client still chooses bank A, if

$t_{A,t} = (T-k)/k\ t_{B,t} + (2k-T)/k\ (h_A^m + m) - \eta,$

Because η is negligibly small, we can ignore it (when k < T-k+1). Thus, subject to condition (1), (a2) transforms into form

(a4)
$$t_{B,t} > \frac{(T-2k)}{(T-k)}(h^m + m) + \frac{k^3}{(T-k)(k+1)^2} t_A^m + \frac{(2k+1)k}{(T-k)(k+1)^2} c_t$$
$$- \frac{k(T+1)}{(T-k)(k+1)^2} \delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}).$$

From (a3) and (a4) it obtained an equilibrium condition under which A captures new clients. Taking into account that $t_A^m = t_B^m \equiv t^m$, this condition can be presented in the form

$$\frac{(2k-T)}{(T-k)}(h^m + m) + [\frac{(T-k)^2}{(T-k+1)^2} - \frac{k^3}{(T-k)(k+1)^2}]t^m + [\frac{2(T-k)+1}{(T-k+1)^2} - \frac{k(2k+1)}{(T-k)(k+1)^2}]c_t$$
$$+ \frac{k(T+1)}{(T-k)(k+1)^2} \delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}) - \frac{T+1}{(T-k+1)^2} \delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}) > 0.$$

After manipulation and after setting $t^m = h^m + m$, the above inequality transforms into the form

(a5)
$$\frac{(2k-T)(3k(T-k)+2T+1)}{(T-k)(k+1)^2(T-k+1)^2}(t^m - c_t)$$
$$+ \frac{k(T+1)}{(T-k)(k+1)^2} \delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}) - \frac{T+1}{(T-k+1)^2} \delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}) > 0.$$

In (a5) the first row is clearly positive insofar as (2k-T) > 0. We then consider the sum terms in the second row. Using equations (11) it can be shown that

(a6)
$$D_{A,t+1}^{k+1} - D_{A,t+1}^{k} = (t^m - c_t) \frac{(2k+1)(1 - \frac{1}{T+1})^2}{(T+1)[1 - (1 - \frac{1}{T+1})^2 \delta]}$$

and that

(a7)
$$D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k} = (t^m - c_t) \frac{(2(T-k)+1)(1 - \frac{1}{T+1})^2}{(T+1)[1 - (1 - \frac{1}{T+1})^2 \delta]},$$

from which follows that the second row of (a5) can be presented in the form

(a8)
$$[\frac{k}{(T-k)(k+1)^2}(2k+1) - \frac{1}{(T-k+1)^2}(2(T-k)+1)]\frac{\delta(1-\frac{1}{T+1})^2(t^m-c_t)}{1-(1-\frac{1}{T+1})^2\delta}.$$

The above expression is positive, if

(a9)
$$(2k-T)[3k(T-k)+2T+1]>0,$$

which shows that only requirement 2k-T > 0 is needed. So we have shown that inequality (a5) also is valid when 2k-T > 0, which completes the first stage of the proof.

The second stage of the proof

The left hand side of expression (a5) tells the difference $t_{B,t}^I$ - $t_{B,t}(t_{A,t}^I)$ when the future prices are at the level $t_{A,t+h}^I$ and $t_{B,t+h}^I$ (with h > 0). By multiplying this term by (T-k)/k, it is obtained from (a5) for the difference $t_{A,t}(t_{B,t}^I)$ - $t_{A,t}^I$ ($\equiv \Delta t_{A,t}$) an expression

(a10)
$$\frac{(2k-T)(3k(T-k)+2T+1)}{k(k+1)^2(T-k+1)^2}(t^m-c_t) + \frac{(T+1)}{(k+1)^2}\delta(D_{A,t+1}^{k+1}-D_{A,t+1}^{k}) - \frac{(T-k)(T+1)}{k(T-k+1)^2}\delta(D_{B,t+1}^{T-k+1}-D_{B,t+1}^{T-k})>0,$$

when $D_{A,t+1}^{k+1}$- $D_{A,t+1}^{k}$ is given in (a6) and $D_{B,t+1}^{T-k+1}$- $D_{B,t+1}^{T-k}$ in (a7). Taking into account equations (a6) and (a7), we see from (a10) that $\Delta t_{A,t}$ increases in the number of A's old clients in so far as k > T-k. Therefore $\Delta \mathbf{t_A}^* \geq \Delta \mathbf{t_A}$. In the considered setting, where $V_{A,t+1}$ describes the present net profits from a strategy in which A captures all the future generations $V_{A,t+1}$ is increasing in the number of the clients so that a given rise in $t_{A,t+h}$ increases $V_{A,t+1}((k+1)(1- 1/(T+1)), \mathbf{t_A}^{I*})$ more than $V_{A,t+1}(k(1- 1/(T+1)),\mathbf{t_A}^{I})$. By the definition given above, $D_{A,t+1}^{k+1}$ = $V_{A,t+1}((k+1)(1- 1/(T+1)), \mathbf{t_A}^{I*})$ and $D_{A,t+1}^{k}$ = $V_{A,t+1}(k(1- 1/(T+1)),\mathbf{t_A}^{I})$. Because $\Delta \mathbf{t_A}^* \geq \Delta \mathbf{t_A}$, this effect even strengthens. From this follows that $V_{A,t+1}((k+1)(1-1/(T+1)),\mathbf{t_A}^{*})$ - $V_{A,t+1}(k(1- 1/(T+1)),\mathbf{t_A})$ > $V_{A,t+1}((k+1)(1- 1/(T+1)),\mathbf{t_A}^{I*})$ - $V_{A,t+1}(k(1- 1/(T+1)), \mathbf{t_A}^{I}))$ = $D_{A,t+1}^{k+1}$ - $D_{A,t+1}^{k}$ in the considered setting. This says that condition (12) is valid and that A's decision to capture new clients in current period is confirmed. The original result, according to which, in an equilibrium it is A who captures the new clients and B who releases, is thus valid.

Finally we consider is it sufficient for bank A to set $t_{A,t}^{o}$ so that $t_{B,t}(t_{A,t}^{o}) = t_{B,t}^{I} - \eta$, so long as it is bigger than B, to guarantee that B will never consider the capture. B's prospects depend very much on the behaviour in symmetry where A and B are about of the same size. It is remarkable that in the symmetry (or near the symmetry) both would compete the prices offers down to indifference levels $t_{A,t}^{I}$ and $t_{B,t}^{I}$, if they would take it as given that each of them could credibly keep the rival indifferent between the capture and release in further periods. In that case

$$V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A}^*) - V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A}) = D_{A,t+1}^{k+1} - D_{A,t+1}^{k}$$

$$V_{B,t+1}((T-k+1)(1 - 1/(T+1)),\mathbf{t_B}^*) - V_{B,t+1}((T-k)(1 - 1/(T+1)),\mathbf{t_B}) = D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}.$$

But, if the banks see that by bidding under the rival they can become bigger and seize on possibly bigger profits, the pricing in symmetry would alter. Given $t_{B,t}$, bank A has in symmetry an incentive to bid lower until $t_{B,t}$ is so low that B gets at the most $D_{B,t+1}^{T-k}$, when A does not choose to bid lower so that B captures the future generations. If both banks make low bids they suffer, however, remarkable losses. Thus in the symmetry there is no stable equilibrium. The possibility of price war does not increase the banks' expected net present values in the symmetry. On the contrary, the possibility that both banks would bid under indifference levels would make the concerning net present value even smaller. Thus we regard that in symmetry (when $T-k \approx T/2$), B's net present values is at the highest $D_{B,t+1}^{T-k}$.

In fact, in the symmetry both banks can make its rival indifferent between a capture and release, and at least one of them can be made indifferent from now on. The bank, which lets its rival to capture the new clients in the future, must in the symmetry be equally well-off as its rival. From this follows logically that the value of neither bank can in the symmetry be above D-function.

Consider then a situation in which $k > T-k$, and in which B captures new generations (during i periods) until it is of equal size with A. A is assumed to set $t_{A,t}^{o}$ so that $t_{B,t}(t_{A,t}^{o}) = t_{B,t}^{I} - \eta$ and η is negligible small. Because B's expected net present value after i periods is at the highest $D_{B,t+1}^{G}$, where $G = (T-k+1)(1-(1/(T+1))^{i} + (1-(1/(T+1))^{i-1} + \ldots + (1-(1/(T+1))$, B's cumulated losses from capturing new generations are, however, then bigger than the profits gained in the form $\delta^{i} D_{B,t+1+i}^{G} - D_{B,t+1}^{T-K}$.

Proof of corollary 1.

From the proof of proposition 1 appears that when bank A captures new clients, bank A should set $t_{A,t}$ so that $t_{B,t}$ is slightly below the threshold $t_{B,t}^{I}$ expressed on the right hand side of (a3). This conditions is

$$t_{B,t} < \frac{(T-k)^2}{(T-k+1)^2} t_B^m + \frac{2(T-k)+1}{(T-k+1)^2} c_t - \frac{T+1}{(T-k+1)^2} \delta (D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}).$$

Taking into account that A should set $t_{A,t}$ also subject to condition (1) and make thus sure that new clients choose bank A in the current period, the above inequality can be written as

$$t_{A,t} < \frac{2k-T}{k}(h_A^m + m) + \frac{(T-k)^3}{k(T-k+1)^2} t^m + \frac{2(T-k)+1}{k(T-k+1)^2} c_t - \frac{(T+1)(T-k)}{k(T-k+1)^2} \delta (D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}).$$

Above

$$D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k} = (t^m - c_t) \frac{(2(T-k)+1)(1-\frac{1}{T+1})^2}{(T+1)(1-(1-\frac{1}{T+1})^2)\delta}.$$

(see, the proof of proposition 1). Inserting this into the above expression of $t_{A,t}$ and setting $t^m = h^m + m$, gives the expression in corollary 1.

The proof of proposition 2

We consider a model where bank A who has k old clients can earn at each period at the most $k(t^m - c_t)$. Because there are no network externalities, the new clients choose A, if $t_{A,t} < t_{B,t}$. We show that there cannot exist an equilibrium in which it pays for A to capture new generations and in which it is not profitable for B to capture new generations. Suppose A has originally k generations. It is profitable for A to capture new clients, if (b1) $$t_{A,t} > \frac{k}{(k+1)}(t_A^m - c_t) + c_t - \frac{1}{(k+1)}\delta(V_{A,t+1}^{k+1} - V_{A,t+1}^{k}).$$

In (b1) $V_{A,t+1}^{k+1}$ is A's net present value in period t+1 when it had (k+1)(1- 1/(T+1)) old generations in that period. Respectively in $V_{A,t+1}^{k}$ A has k(1- 1/(T+1)) old generations. When it does not pay for B to capture new generations, it is required

(b2) $$t_{B,t} < \frac{(T-k)}{(T-k+1)}(t_B^m - c_t) + c_t - \frac{1}{(T-k+1)}\delta(V_{B,t+1}^{T-k+1} - V_{B,t+1}^{T-k}).$$

Inequalities (b1) and (b2) imply that it is profitable for A to capture new clients so long as

(b3) $$-\frac{(2k-T)(t^m - c_t)}{(k+1)(T-k+1)} - \delta\frac{(V_{B,t+1}^{T-k+1} - V_{B,t+1}^{T-k})}{(T-k+1)} + \delta\frac{(V_{A,t+1}^{k+1} - V_{A,t+1}^{k})}{(k+1)} > 0.$$

The left hand side of (b3) is $t_{B,t}^I$- $t_{B,t}(t_{A,t}^I)$. Suppose that A captures all the new generations from now on. It pays for A to set prices so that B will be almost indifferent between the capture and release. In that case we can replace $V_{B,t+1}^{T-k+1}$ for $V_{B,t+1}((T-k+1)(1 - 1/(T+1)),\mathbf{t_B^{I*}})$ = $D_{B,t+1}^{T-k+1}$, and $V_{B,t+1}^{T-k}$ for $V_{B,t+1}((T-k)(1 - 1/(T+1)),\mathbf{t_B^{I*}})$ = $D_{B,t+1}^{T-k}$. Then $V_{B,t+1}^{T-k+1}$ - $V_{B,t+1}^{T-k}$ in (b3) is the same as $D_{B,t+1}^{T-k+1}$ - $D_{B,t+1}^{T-k}$ which has an equation

(b4) $$\frac{(1-\frac{1}{T+1})}{1-(1-\frac{1}{T+1})\delta}.$$

Suppose that also A is originally indifferent between capture and release. Then also $V_{A,t+1}^{k+1}$ - $V_{A,t+1}^{k}$ can be substituted for expression (b4). Using expression (b4), the left hand of inequality (b3) becomes

(b5) $$-\frac{(2k-T)(t^m - c_t)}{(k+1)(T-k+1)}[1+\frac{(1-\frac{1}{T+1})}{1-(1-\frac{1}{T+1})\delta}].$$

Expression (b5) is negative which shows that A can not capture new generations by profit.

Because there are no network externalities $t_{A,t}^I$ = $t_{B,t}(t_{A,t}^I)$ and $t_{B,t}^I$ = $t_{A,t}(t_{B,t}^I)$. The expression (b5) describes $t_{B,t}^I - t_{A,t}^I$ on the condition that both banks are indifferent between the capture and the release in the future. This expression is negative which refers to the fact that $t_{B,t}^{I} < t_{A,t}^{I}$ insofar as both are indifferent in the future. Suppose that A decides however to capture new clients in the current period. Then it should lower its prices below $t_{A,t}^{I}$ to the level $t_{A,t} = t_{B,t}^{I}$. If A also behaves in this way consistently in future, its future prices $t_{A,t+h}^{I}$ are replaced by prices $t_{A,t+h}$ (= $t_{B,t+h}^{I}$). This means that $V_{A,t+1}^{k+1}$ - $V_{A,t+1}^{k}$, which was originally $V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A^{I*}})$ - $V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A^{I}})$ (= $D_{A,t+1}^{k+1}$ - $D_{A,t+1}^{k}$) becomes $V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A}^*)$ - $V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A})$ when $\mathbf{t_A}^{*\mathbf{I}} = t_{A,t+1}^{I*}, t_{A,t+2}^{I*}, t_{A,t+3}^{I*}, ...$, $\mathbf{t_A} = t_{A,t+1}^{I}, t_{A,t+2}^{I}, t_{A,t+3}^{I}, .. ,$ $\mathbf{t_B}^* = t_{B,t+1}^{I*}, t_{B,t+2}^{I*}, t_{B,t+3}^{I*}, ...$, and $\mathbf{t_B} = t_{B,t+1}^{I}, t_{B,t+2}^{I}, t_{B,t+3}^{I}, ...$ Because $t_{B,t}^{I} - t_{A,t}^{I}$ in (b3) decreases in k (number of A's old generations), it is clear that also $t_{B,t+h}^{I} - t_{A,t+h}^{I}$ decreases in the number of A's old generations when h ≥1. This means that $\mathbf{t_A}^* - \mathbf{t_A}^{I*} < \mathbf{t_A} - \mathbf{t_A}^{I}$. Because each $t_{A,t+h}^{I*}$ is lowered more than each $t_{A,t+h}^{I}$ (when h ≥1), the difference $V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A}^*)$ - $V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A})$ becomes smaller than $V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{t_A^{I*}})$ - $V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{t_A^{I}})$ even when net present values describe the situation in which A captures all the future generations. This only confirms that inequality (b3) cannot be true when A captures new clients.

The proof of proposition 3.

The proof of corollary 1 follows the proof of proposition 1. Using equations (13) and (14) instead of equations (3) and (7), the conditions (a3) and (a4) turn into form

(c1)
$$t_{B,t} < -\frac{(T-k)}{(T-k+1)}t_B^m + \frac{2(T-k)+1}{(T-k+1)}c_t - \frac{T+1}{(T-k+1)}\delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}).$$

and

(c2)
$$t_{B,t} > \frac{(T-2k)}{(T-k)}(h^m + m) - \frac{k^2}{(T-k)(k+1)}t_A^m + \frac{(2k+1)k}{(T-k)(k+1)}c_t - \frac{k(T+1)}{(T-k)(k+1)}\delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}).$$

From (c1) and (c2) follows the requirement

$$\frac{(2k-T)}{(T-k)}(h^m+m)+[\frac{k^2}{(T-k)(k+1)}-\frac{(T-k)}{(T-k+1)}]t^m+[\frac{2(T-k)+1}{(T-k+1)}-\frac{(2k+1)k}{(T-k)(k+1)}]c_t$$
$$+\frac{k(T+1)}{(T-k)(k+1)}\delta(D_{A,t+1}^{k+1}-D_{A,t+1}^{k})-\frac{T+1}{(T-k+1)}\delta(D_{B,t+1}^{T-k+1}-D_{B,t+1}^{T-k})>0,$$

in which $t_A^m = t_B^m \equiv t^m$. Next we insert $t^m = h^m + m$ into the above inequality and manipulate the terms after which it transforms into

(c3)
$$\frac{(2k-T)(2k(T-k)+2T+1)}{(T-k)(k+1)(T-k+1)}(t^m-c_t)$$
$$+\frac{k(T+1)}{(T-k)(k+1)}\delta(D_{A,t+1}^{k+1}-D_{A,t+1}^{k})-\frac{T+1}{(T-k+1)}\delta(D_{B,t+1}^{T-k+1}-D_{B,t+1}^{T-k})>0.$$

The sum of the terms in the first row of (c3) is positive insofar as 2k-T > 0. In the second row $D_{A,t+1}^{k+1} - D_{A,t+1}^{k} > D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}$, from which follows directly that the sum of the terms in the second row is positive also so long as 2k-T > 0. This proves that (c3) is valid when k > T-k.

Following the proof of proposition 1, we should also prove that $\Delta \mathbf{t}_A^* \geq \Delta \mathbf{t}_A$ which guarantees that condition (12) is valid, in so far as 2k-T > 0. Comparing (c3) with (a5), it is clear, on the basis of the proof of proposition 1, that this requirement is met.

The proof of proposition 4.

We show again that that there exists such an credible price offer $h_{A,t}^o$ which makes the client to choose bank A in period t subject to condition $W_{A,t}(k) - X_{A,t}(k) > 0$ and $X_{B,t}(T-k) - W_{B,t}(T-k) < 0$.

The first stage of the proof

Following the proof of proposition 1, we assume that the net present values in period t+1 are $D_{A,t+1}^{k+1}$, $D_{A,t+1}^{k}$, $D_{B,t+1}^{T-k+1}$ and $D_{B,t+1}^{T-k}$. Then it is profitable for bank A to capture new clients, if

(d1)
$$h_{A,t}>c_h-\frac{2k+1}{(T-k)(k+1)}(t^m-c_t)-\frac{T+1}{(T-k)(k+1)}\delta(D_{A,t+1}^{k+1}-D_{A,t+1}^{k}).$$

Let us assume that bank A sets $h_{A,t}$ only slightly below the level in which the client is indifferent between A and B. Taking this and condition (15) into account, (d1) can be expressed in terms of $h_{B,t}$ by

(d2) $$h_{B,t} > -\frac{2k-T}{k}m + \frac{T-k}{k}c_h + \frac{2k+1}{k(k+1)}c_t + \frac{k(2k-T)-T-1}{k(k+1)}t^m - \frac{(T+1)}{k(k+1)}\delta(D^{k+1}_{A,t+1} - D^{k}_{A,t+1}).$$

The condition under which it is unprofitable for B to capture the new clients, is now

(d3) $$h_{B,t} < c_h - \frac{2(T-k)+1}{k(T-k+1)}(t^m - c_t) - \frac{(T+1)}{k(T-k+1)}\delta(D^{T-k+1}_{B,t+1} - D^{T-k}_{B,t+1}).$$

Taking into account assumptions $t^m = h^m + m$ and $h^m = c_h$ and combining of conditions (d2) and (d3), yields

(d4) $$\frac{(2k-T)}{k(k+1)(T-k+1)}(t^m - c_t) \\ + \frac{(T+1)}{k(k+1)}\delta(D^{k+1}_{A,t+1} - D^{k}_{A,t+1}) - \frac{T+1}{k(T-k+1)}\delta(D^{T-k+1}_{B,t+1} - D^{T-k}_{B,t+1}) > 0.$$

The first row in (d4) is positive. Using expressions (a6) and (a7), the second row of (d4) can be expressed in the form

(d5) $$\frac{2k-T}{k(T-k+1)(k+1)}\frac{\delta(t^m - c_t)}{1-(\frac{T}{T+1})^2\delta},$$

which is clearly positive, if 2k-T > 0.

The second stage

At the second stage we should prove also that $\mathbf{\Delta h_A}^* \geq \mathbf{\Delta h_A}$ when $\mathbf{\Delta h_A}^*$ and $\mathbf{\Delta h_A}$ are similarly defined as $\mathbf{\Delta t_A}^*$ and $\mathbf{\Delta t_A}$ above. This is sufficient for the condition

$$V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{h_A}^*) - V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{h_A}) >$$

$$V_{A,t+1}((k+1)(1 - 1/(T+1)),\mathbf{h_A}^{\mathbf{I}*}) - V_{A,t+1}(k(1 - 1/(T+1)),\mathbf{h_A}^{\mathbf{I}})$$

to be fulfilled. To show that $\Delta h_{A,t}$ (= $h_{A,t}(h_{B,t}^I) - h_{A,t}^I$) is increasing in the number of A's old clients, we form $\Delta h_{A,t}$ by multiplying expression (d4) by k/(T-k). This yields

$$\frac{(2k-T)}{(T-k)(k+1)(T-k+1)}(t^m - c_t)$$
$$+\frac{(T+1)}{(T-k)(k+1)}\delta(D_{A,t+1}^{k+1} - D_{A,t+1}^{k}) - \frac{T+1}{(T-k)(T-k+1)}\delta(D_{B,t+1}^{T-k+1} - D_{B,t+1}^{T-k}) > 0.$$

The first row clearly increases in k when expressions $D_{A,t+1}^{k+1}$ - $D_{A,t+1}^{k}$ and $D_{B,t+1}^{T-k+1}$ - $D_{B,t+1}^{T-k}$ have equations (a6) and (a7). In the above equation the second row transforms into

$$\frac{(2k-T)}{(T-k)(T-k+1)(k+1)}[2(1-\frac{1}{T})-1]\frac{\delta(t^m - c_t)}{1-(\frac{T}{T+1})^2\delta},$$

which clearly increases in k. The rest of the proof relies on the second stage of the proof of proposition 1.