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**PEKKA KORPINEN**

**A MONETARY MODEL OF LONG CYCLES**

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## ABSTRACT

The explanations of the long cycle phenomenon in the real economy is searched in the paper by introducing monetary factors into a neo-Keynesian type of growth model a la Harrod, Kaldor and Kalecki. A cumulatively unstable growth process is turned into a long cycle through monetary intervention: during a long upswing, the monetary policy stance, which is an endogenous variable in the model, is turning less and less accommodative along with accelerating inflation. "Sadistic" monetary squeeze finally turns the long upswing into a long downswing with increasing unemployment, disinflation and finally deflation. The simpler version of the model makes use of the quantity equation of money and nominal interest rate to link the monetary sector to the real economy. In the more complicated model variant, the effects of monetary policy are channelled into the real economy via real interest rates and investments. Both model variants are based on non-linear differential equations, the solutions of which utilize either the known properties of the Lotka-Volterra system or so-called Liapunov's first method for almost linear systems.

# A MONETARY MODEL OF LONG CYCLES\*

Pekka Korpinen

The World Bank and

Labour Institute for Economic Research, Finland

## 1. INTRODUCTION

The reasons for slow economic growth since 1973 are not yet fully understood. In search of possible explanations also the long cycle hypothesis has attracted renewed interest.

Originally, long term fluctuations were first observed in price levels and other monetary variables (Tooke and Newmarch 1928, Jevons, 1887, Wicksell, 1936, Cassel, 1932). Kondratieff (1935) was one of the first to integrate monetary and real factors into a general long cycle hypothesis. His preliminary theoretization efforts concentrated around long term fluctuations in the price and interest rate levels, loanable funds and long term investments (Garvy, 1943, pp. 207-208). In contrast to Kondratieff, Schumpeter (1939) layed the foundation for the theory of long cycles based on innovations and technical change leaving monetary factors to a secondary place.

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I think it is correct to say that the majority of the contemporary long cycle/wave-researchers (e.g. Mench, 1975, Mandel, 1975, Forrester, 1976, Rostow, 1978, Freeman, 1981, and van Duijn, 1983) follow Schumpeter in the sense that key explanatory factors lay outside the monetary field.

In this paper (see also Korpinen, 1981), reasons for long term fluctuations in economic activity and employment are looked up in the first place from changes in the price level, monetary policy and interest rates. A model will be so specified that through introduction of monetary variables a Harrod-type balanced growth path can be turned into long term oscillations around the moving equilibrium.

The motivation to concentrate on monetary factors comes among other things, from the following observations. In the 1970s there was a shift towards a less accommodative monetary policy stance in several major industrial countries, especially in the United States. The following increase in nominal and real interest rates has led to distortions in debt-equity ratios and debt crises for nations, governments, important sectors such as agriculture and energy and large firms of "old industries". For the most part the adjustment takes the form of reduced investment expenditure. High interest rates and unemployment have also led to rapid disinflation and in some cases, e.g. in commodity prices, to a straight deflation.

From the historic point of view, the real interest rates can stay high for a considerable time. As a matter of fact, real interest rates have followed during the last two hundred years the pattern of long

cycles amazingly well (see Shiller and Siegel, 1977). Thus, it is not unreasonable to have real interest rate among the key explanatory variable of the long cycles.

## 2. THE MODEL

Following Harrod (1939) we assume that in equilibrium output ( $Y$ ) grows ( $\dot{Y}$ ) a constant fraction ( $1/k$ ) of incremental capital ( $I$ ). It is further assumed that wages ( $w$ ) are consumed and profits ( $x$ ) are invested. If labour force is constant and productivity ( $Y/N$ ) grows at a constant rate ( $\dot{y}$ ), steady growth is warranted when  $x = k\dot{y}$ .

What would happen, if the Harrod type growth process gets, for one reason or another, out of balance? In order to answer that question we need a set of equations describing basic behavioural relationships outside the equilibrium path.

We will start with the demand and supply of money. We assume that the velocity of money is inversely proportional to the interest rate:

$$(1) \quad M_d = \frac{\alpha YP}{1 + \beta(R - \dot{y})},$$

where  $M_d$  is the demand for money,  $Y$  is national income in real terms,  $P$  is price level,  $R$  is interest rate,  $\dot{y}$  is rate of growth of productivity (a constant, which is equal to "natural rate of interest"), and  $\alpha$  and  $\beta$  are positive coefficients.

The rate of accommodation of the money supply depends on the wage inflation.

$$(2) \quad M_s = \frac{\alpha YP}{1 + \gamma(\hat{w} - \dot{y})},$$

where  $\hat{w}$  is the relative increase of money wages ( $\dot{w}/w$ ), and  $\gamma$  is a positive coefficient.

The supply and demand of money determine the interest rate.

$$(3) \quad R = A + B\hat{W},$$

where  $A = (1 - \frac{\gamma}{\beta})\hat{y}$  and  $B = \frac{\gamma}{\beta}$ .

The equilibrium condition for the market of goods and services is

$$(4) \quad Y = C + I,$$

where  $C$  is the volume of consumption and  $I$  is the volume of investment.

The consumption function is based on a notion (à la Kalecki) that "workers spend what they get and capitalists get what they spend".

$$(5) \quad C = wN,$$

where  $w$  is the real wage and  $N$  is employment as well as employment rate, because labour force is assumed to be constant and the measurement of labour force is normalized to unity.

The volume of investment is assumed to depend on the growth of output, profits and interest rate.

$$(6) \quad I = k\dot{Y} + \delta(\Pi - k\hat{y}Y) - \zeta(R - \hat{y})Y,$$

where  $k$  is capital-output ratio,  $\Pi$  is real profits (equal to  $I$ ),  $k\hat{y}Y$  is "natural" real profits, and  $\delta$  and  $\zeta$  are positive coefficients.

The growth rate of nominal wages depends on the growth of productivity, the share of profits (or wages) and the change in employment.

$$(7) \quad \hat{W} = \hat{y} + \eta(x - k\hat{y}) + \nu\hat{N},$$

where  $x$  is the share of profits,  $k\hat{y}$  is "natural" share of profits, and  $\eta$  and  $\nu$  are positive coefficients.

The rate of inflation is determined by the unit labour costs and the change in mark-up factor, which depends on the capacity utilization ratio described by the employment rate.

$$(8) \quad \hat{P} = \hat{W} - \hat{y} + \alpha(N - \tilde{N}),$$

where  $\kappa$  and  $\tilde{N}$  are positive coefficients.

From equations (7) and (8) we get

$$(9) \quad \dot{z} = \kappa (\tilde{N} - N),$$

where  $\dot{z}$  is the relative change of the share of wages, ( $x = 1 - z$ ).

Equations (3) - (7) lead to

$$(10) \quad \dot{\tilde{N}} = D(k\hat{y} - 1 + z),$$

$$\text{where } D = \frac{\delta - 1 - \beta \eta}{k - \beta \nu}.$$

Equations (9) and (10) form a nonlinear Lotka-Volterra system, which has an equilibrium point  $(\tilde{N}, 1 - k\hat{y})$ . If the system is in equilibrium both the capital stock and output grows along the Harrod's warranted growth path. In this case capital, output and productivity all have the same growth rate ( $\hat{y}$ ).

### 3. ON THE PROPERTIES OF THE MODEL

In order to analyze the properties of the model we first solve the equation of the trajectories of the system (9) - (10). Eliminating time and performing integration we get

$$(11) \quad \kappa \tilde{N} \log N - \kappa N + (D - Dk\hat{y}) \log z - Dz = \log c,$$

where  $c$  is an integration constant.

Equation (11) can also be written

$$(12) \quad \frac{z^D (1 - k\hat{y})}{e^{Dz}} \cdot \frac{N^{\kappa \tilde{N}}}{e^{\kappa N}} = c.$$

Equation (12) describes the trajectories (given the initial condition), which form a closed curve, if  $D > 0$ .  $N$  and  $z$  reach their maximum and

minimum values on the trajectories when  $z$  is  $1 - k\bar{y}$  and  $N$  is  $\bar{N}$ . The movement along the trajectories is counterclockwise.

If  $D < 0$ , then the equilibrium point is not a center, but on "the knife edge": disturbances lead either to cumulative inflation and full employment or deflation and increasing unemployment.

The numerator of  $D$  is always negative, as  $\delta$  has to be smaller than one. Therefore the sign of  $D$  depends whether  $k \gtrless \frac{1}{\beta v}$ . One could assume, that capital-output ratio ( $k$ ) is somewhere around, say 4. As the values of  $\beta$  and  $v$  are probably somewhere around one, the value of  $B$  has to be quite high in order to turn a unstable growth process into a cyclic one.

The value of  $B$  depends on how sensitive money supply is to wage inflation as well as on how inelastic the velocity of money is in relation to interest rate. It is only through "sadistic" monetary policy, which lets interest rates skyrocket that the cumulatively inflationary long upswing phase is turned in the model to disinflation and finally deflationary long downswing.

The next property of the model to be analyzed is the length of the cycle. Even though there also exists an explicit solution (Frame, 1974) to the length of the cycle in the Lotka-Volterra systems, there is an easy way to get approximate solution using a linearized version of the model.

We can transfer the equilibrium point of the system

$$(13) \quad \begin{aligned} \dot{N} &= D (k\hat{y} - 1 + z)N \\ \dot{z} &= \lambda (\tilde{N} - N)z \end{aligned}$$

to to origin by letting  $n = N - \tilde{N}$  and  $v = z - 1 + k\hat{y}$ ,

$$(14) \quad \begin{aligned} \dot{n} &= D\tilde{N}v + Dvn \\ \dot{v} &= -\lambda(1 - k\hat{y})n - \lambda nv \end{aligned}$$

When only the linear terms of (14) are taken into account, the solution of the system is

$$(15) \quad \begin{aligned} n &= c_1 \cos \sqrt{DN\lambda(1 - k\hat{y})} t + c_2 \sin \sqrt{DN\lambda(1 - k\hat{y})} t \\ v &= c_3 \cos \sqrt{DN\lambda(1 - k\hat{y})} t + c_4 \sin \sqrt{DN\lambda(1 - k\hat{y})} t. \end{aligned}$$

The length of the cycle (T) can be calculated from the formula

$$T = \frac{2\pi}{\sqrt{DN\lambda(1 - k\hat{y})}}$$

It is very difficult to say a priori anything specific about the quantitative values of some of the parameters. In order to reach the range of 40 to 60 years, acceleration of inflation has to be a slow process, i.e. both  $\eta$  and  $\lambda$  should be very small. In practice this means that the workers are slow to increase wage demands after the share of profits has increased and that capitalists are slow to increase their mark-up over the unit costs when the demand is strong. 20 to 30 years as a length of an inflationary phase is not necessarily in contradiction with empirical observations before the First and after the Second World War. By assuming the pricing to react slowly one is in a sense eliminating the short cycles and describes only the long term fluctuations in the price level. It is not excluded that a similar model structure with

different parameter values could be used to describe the so called stop-go-cycle as well.

#### 4. A FURTHER DEVELOPMENT

The initial conditions of the model affect the amplitude of the cycle, but not the average values over time of employment ( $N$ ) and distribution of income ( $z$ ). On empirical grounds one can question (see Korpinen, 1981, p. 152) the fact that the cycle is symmetric to the axis in a sense that the two values of employment assigned to each value of the share of wages always have the same average, i.e. the equilibrium value. One would, perhaps, want to see a trade-off between employment and a share of wages (profits). If wages are pushed to their maximum, where there are no profits, one would expect that employment is below the average.

In the following we will achieve this result, but at the cost of complication of the model.

By using real interest rate ( $R - \hat{P}$ ) instead of nominal interest rate in the investment function (6), the model can be described by equations

$$(9) \quad z = \alpha (\tilde{N} - N)$$

$$(10') \quad \tilde{N} = F + Gz + HN$$

where  $F$  is 
$$\frac{[1 + (B - 1)\{\eta - \delta\} (1 - ky) + \xi \alpha \tilde{N}}{k + (1 - B)\xi v},$$

$$G \text{ is } \frac{- [1 + (B - 1)\{\eta - \delta\}]}{k + (1 - B)\xi v},$$

and  $H$  is 
$$\frac{- \xi \alpha}{k + (1 - B)\xi v}$$

Equations (9) and (10) form a dynamic, almost linear system (prove see Korpinen, 1981, p. 148) which has an equilibrium point  $(\tilde{N}, 1 - k\tilde{y})$ , i.e. as in the model (9) - (10). With the same reasoning about the magnitude of the parameters as in the previous model, the system can be proved, using Liapunov's first method (Boyce and DiPrima, 1969, pp. 394-95), to be an explosive spiral (Figure 1).

## 5. CONCLUDING REMARKS

Even though the monetary variables play a key role in the model, it is not correct to call it monetarist. It is even difficult to speak about the effects of monetary policy as the money supply is totally endogeneous in the model.

In order to try to give a causal interpretation, let us call a period with high interest rate a "monetarist" regime and the period with low interest rate a "Keynesian" regime. The monetary squeeze during the monetarist regime works its effect to price level through investment, employment and distribution of income. Disinflation or deflation is gained only by substantial and long lasting losses in production.

The Keynesian era is characterized by low interest rates, increasing employment and rising share of profits, which lead to militancy and greater demands by workers. The ongoing class struggle has a cumulative momentum of its own. Inflation accelerates and the tone of monetary "policy" starts to change.

The price and wage mechanism, described by the model, refers to a closed economy, i.e. to the world as a whole. If world-market prices

were given, as in the case of a small open economy with fixed exchange rates, then higher nominal wages should lead to higher real wages (as postulated in Goodwin's (1967) model, which in a sense is the opposite case of our model). Without "given" outside markets, prices are basically determined by costs, out of which the wages are the most important part.

The distribution assumption in the foregoing analysis is similar to Kaldor's (1955-56) growth model, in which the relation between prices and money wages depends on the total demand (investment). The idea here is that perpetual high employment increases the degree of monopoly (prices in relation to price costs, see Kalecki, 1943). In the normal business cycle recession the degree of monopoly might continue to increase temporarily as overhead costs are relatively higher, but during the long depression phase unused capacity will eventually bring a halt to the monopolistic pricing process culminating in cutthroat competition.

Moving outside the model, it is plausible that the economic policy has a tendency to be late and therefore "wrong". If the reality determines the consciousness and the consciousness determines the action, there is an unavoidable lag. The writings of Smith (1776) and Ricardo (1817) were made during an inflationary long upswing, but their recommendations (hard currency, laissez-faire) were still applied during a deflationary period. Tooke (1826-57) and Keynes (1936) made their contributions during a deflationary long downswing, but their policy recommendations were mainly followed in an inflationary environment.

This leads to the theory of long cycles in its simplest form: the same mistakes are repeated by every second generation. The economic policy of the 1970s and 1980s resembles surprisingly that of the 1920s and 1930s as well as that of the 1870s and 1880s or even further back, that of the 1820s and 1830s.

The emphasis on monetary factors in this paper does not imply that innovations and other technological factors are unimportant. It is only too difficult to analyze all the relevant variables simultaneously. It may very well be, that monetary factors are relatively more important in explaining the turning point from the long upswing into the long downswing whereas the cumulation of unused technological possibilities might have comparative advantage in explaining the resumption of growth.

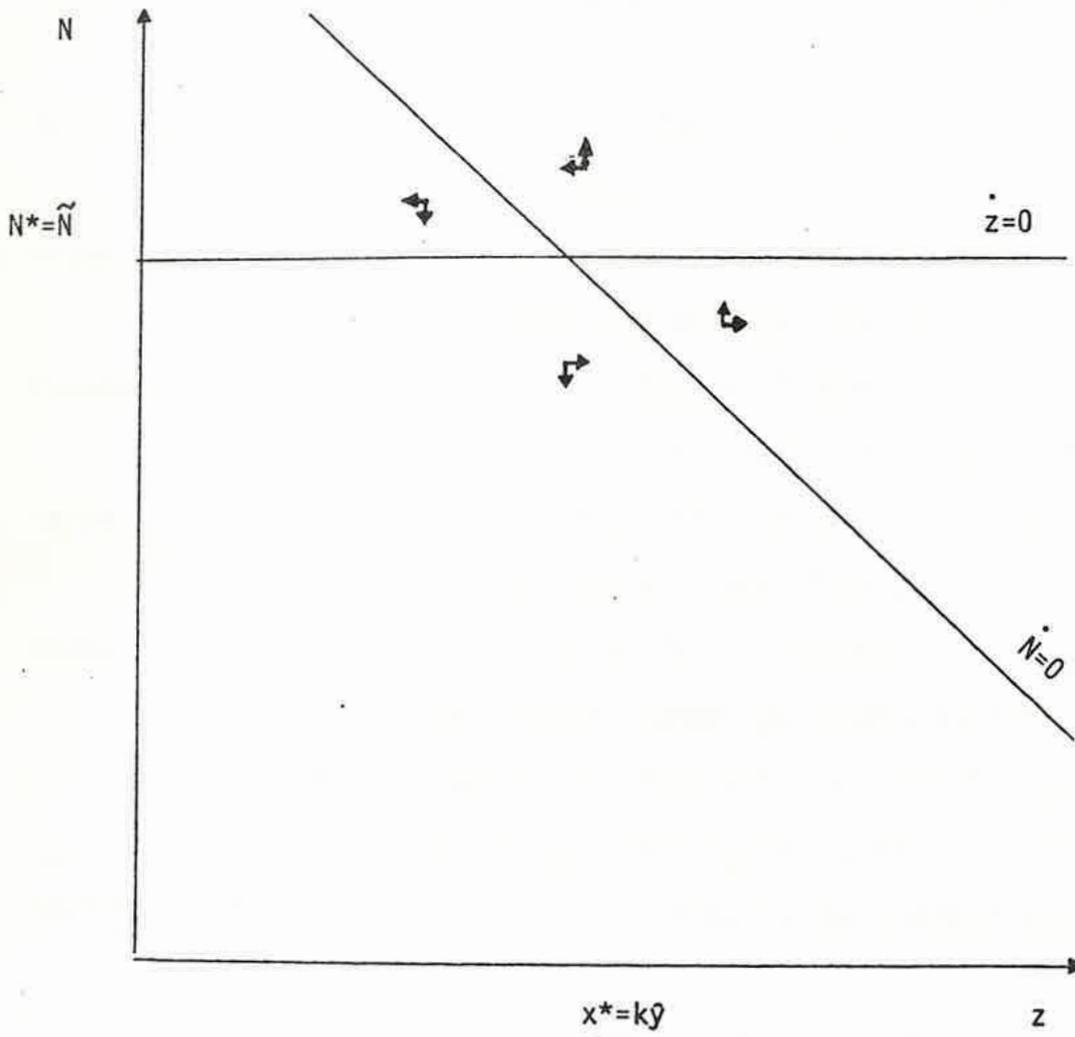


Figure 1

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