TYÖVÄEN
TALOUDELLINEN
TUTKIMUSLAITOS

TUTKIMUSSELOSTEITA 25

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EXCHANGE RATE POLICY RULES: A NOTE

HELSINKI 1983

TYÖVÄEN TALOUDELLINEN TUTKIMUSLAITOS

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June 1983

ISBN 951-9281-37-1

ISSN 0357-9603

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EXCHANGE RATE POLICY RULES: A NOTE

1. Introduction

Exchange rate policy has been one of the most lively discussed issues in the Finnish debate on economic policy. The discussion of exchange rate policy rules or norms which define the criteria according to which exchange rate policies could be pursued has become one important part of the debate.

The main contributions to this discussion have been given by Korkman, who provides a theoretical analysis of three alternative rules for exchange rate policy: the fixed exchange rate rule, the inflation norm and the competitiveness norm. The fixed exchange rate rule means that an index of prices of foreign exchange is stabilized; the inflation norm means that the domestic currency value of a price index of foreign goods is stabilized and the competitiveness norm means that the real exchange rate is stabilized. (See Korkman (1980) pp. 79 - 80.) In his study, Korkman discusses the usefulness of these rules in neutralizing various exogenous shocks.

Contributions which are based on the use of an econometric model have been presented e.g. in Halttunen (1981) and Halttunen - Korkman (1983). In these works, the authors examine the merits and drawbacks of the alternative rules (above all the fixed exchange rate norm and the inflation norm) by performing simulation experiments i.e. by calculating dynamic multipliers.

On the other hand, when Korkman conducts his theoretical analysis in Korkman (1980) he proceeds in a way, which has become standard: First, he develops an dynamic macro model which is expressed in levels.

Thereafter, he studies some short run comparative statics properties of the model. Having performed these considerations, he investigates the dynamics of the model - stability etc.

We see that the way of conducting the theoretical analysis differs from the way which is used when the empirically oriented studies have been carried out.

We now pose the following question: can we use a way similar to calculating dynamic multipliers with the help of an econometric model when conducting a theoretical analysis by a (non-linear) dynamic macro model? The answer we give in this note is: yes.

The positive answer is given by variational dynamic analysis introduced into economics by Aoki. (See e.g. Aoki, 1981.) The technique of variational dynamic analysis is based on comparing time paths of the economy which lie near some reference paths or states. Variational dynamics thus describe how models behave near their "control" solution time paths. Consequences of changes of macroeconomic policy instruments as well as changes in exogenous circumstances can be evaluated by this method. A very important merit in the use of variational analysis is that besides short-run and long-run effects of e.g. policy instruments also inter-run effects of these instruments can be studied when variational analysis is used. Therefore, e.g. the time patterns of the effects of policy instruments can be analysed.

In this note, we illustrate how one can apply variational dynamic analysis to examining the merits and drawbacks of the three alternative exchange rate policy rules. Thus this note concerns mainly the method of analysing; it adds very little to the economics of exchange rate policy rules. The main objective of this note is to demonstrate that by using variational analysis one can conduct perhaps a more detailed investigation of the exchange rate policy rules than by using the traditional approach.

We performe all considerations by using a rather simple dynamic non-linear macroeconomic model, which is presented in chapter 2. In chapter 3 we utilize this model in analysing the consequences of the policies corresponding to the three alternative rules.

2. The Model

2.1. The Model in the Level Form

In this chapter, we set up a model which is based on a Keynesian model used by Korkman when he examines the consequences of the three alternative norms for exchange rate policy. (See Korkman, 1981, ch. 4.) In Korkman's model there are three goods: two final goods, which are not perfect substitutes but can be substituted in consumption and investment, and one intermediate input. The behaviour of money market is not modelled. Demands for labour and raw materials are specified by Leontief technology. The dynamics of the model is governed by a Phillips curve and an adjustment equation for the volume of exports.

The model we use in this paper is a simplified version of Korkman's model. The main difference between the models is that we don't assume that there is a time lag in the reaction of export volumes to price changes. Therefore the dynamics of our model is one-dimensional. When illustrating the use of variational equations we don't deal with the effects the exchange rate policy pursued has on the balance of payments. Therefore we omit the equations defining the balance of payments. (See Korkman, 1980, equations (4.9) - (4.11), p. 65.) Furthermore, the inflation dynamics is simpler in our model than in the model used by Korkman: we don't have any expectations term in the Phillips curve.

The model can be written in the following form:

(2.1.) $Y = C^D + I^D + G + X$ (equilibrium condition for the domestic commodity),

(2.2.) $C^D = C^D(Y_h, EP_F/P_D)$ (private consumption of the domestic commodity),

(2.3.) $I^D = I^D(Y_f, EP_F/P_D, r)$ (private investment demand for domestic commodity),

(2.4.) $X = X(EP_F/P_D, Y^*)$ (foreign demand for domestic commodity),

(2.5.) $Y_h = WL/P_D + J - T_h$, (disposable real income of households)

(2.6) $Y_f = Y - WL/P_D - J - (EP_R/P_D)R - T_f$ (real profits retained after taxation),

(2.7) $L = aY$

(production technology)

(2.8) $R = bY$

(2.9) $P_D = P_D(W, EP_R, EP_F)$ (price setting equation)

(2.10) $\dot{W}/W = f(L)$ (Phillips curve),

where

Y = gross output of the domestic commodity,
C^D = private consumption of the domestic commodity,
I^D = private investment of the domestic commodity,
X = the volume of exports,
G = government expenditure, (exogenous)
Y_h = household disposable real income,
E = the domestic currency price of foreign currency (exogenous)
P_D = the price of the domestic commodity
P_F = the foreign currency price of the foreign final commodity (exogenous)
P_A = the foreign currency price of the intermediate input (exogenous)
Y_f = retained after tax profits,
r = the domestic rate of interest (exogenous)
Y^* = output of the foreign (final) commodity (exogenous)
W = nominal wages
J = distributed profits (exogenous)
T_h = household taxes (exogenous)
T_f = firm's taxes (exogenous)
L = employment
R = the volume of raw materials imported.

We assume that the functions C^D, I^D, X, P_D and f, which can be non-linear, possess continuous first partial derivatives. Furthermore, we assume that the model (2.1) - (2.10) has a unique long-run equilibrium state $\bar{Y}, \bar{C}^D, \bar{I}^D, \bar{X}, \bar{Y}_h, \bar{Y}_f, \bar{L}, \bar{R}, \bar{P}_d, \bar{W}$, when $G = \bar{G}$, $E = \bar{E}$, $P_F = \bar{P}_F$, $r = \bar{r}$, $Y^* = \bar{Y}^*$, $J = \bar{J}$, $T_h = \bar{T}_h$ and $T_f = \bar{T}_f$.

In his study, Korkman uses a model which is expressed in levels. We, however, don't use the form (2.1) - (2.10) when conducting the analysis. Our purpose is to illustrate how one can make use of variational analysis when investigating the effects of the exchange rate policy rules introduced by Korkman.

2.2. The Model in the Variational Form

When forming the variational equations of (2.1) - (2.10) we need a reference path or point against which to compare. Reference paths are "normal" or "planned" time paths of the economy. They cannot be total fictions. A reference path must be a time path, which the model can exhibit. We choose the long-run equilibrium state as the reference state of the economy against which alternate time paths of the economy resulting from changes in various exogenous factors are compared. In general, however, reference paths need not equal long-run equilibrium states. (For a more detailed discussion see Aoki, 1981.)

Using the notation $x = (X - \bar{X})/\bar{X} = \delta X/\bar{X}$, where $\bar{X}$ is the long-run equilibrium value of X (an endogenous variable) or the value of an exogenous variable associated with the reference solution, and forming the variational equations of (2.1) - (2.10) we obtain (for details, see Aoki, 1981):

$$(2.11) \qquad y = a_1 c^D + a_2 i^D + a_3 x,$$

where

$$a_1 = \bar{C}^D/\bar{Y},\ a_2 = \bar{I}^D/\bar{Y},\ a_3 = \bar{X}/\bar{Y};^1$$

(2.12) $$c^D = a_4 y - a_5 p_D + a_5 p_F + a_5 e,$$

where

$$a_4 = \frac{C_1 \bar{Y}_h}{\bar{C}^D},\ a_5 = \frac{C_2 \bar{P}_F \bar{E}}{\bar{C}^D \bar{P}_D};^2$$

(2.13) $$i^D = a_6 y - a_7 p_D + a_7 p_F + a_7 e,^3$$

where

$$a_6 = \frac{I_1^D \bar{Y}_f}{\bar{I}^D},\ a_7 = \frac{I_2^D \bar{E}\bar{P}_F}{\bar{I}^D \bar{P}_D};$$

(2.14) $$x = -a_8 p_D + a_8 p_F + a_8 e + a_9 y^*,$$

where

$$a_8 = \frac{X_1 \bar{E}\bar{P}_F}{\bar{X}\bar{P}_D},\ a_9 = \frac{X_2 \bar{Y}^*}{\bar{X}};$$

(2.15) $$y_h = -a_{10} p_D + a_{10} w + a_{10} l,^4$$

where

$$a_{10} = \frac{\bar{W}\bar{L}}{\bar{Y}_h \bar{P}_D};$$

(2.16) $$y_f = a_{11} y + (a_{12} + a_{13}) p_D - a_{12} w - a_{12} l - a_{13} p_R - a_{13} r - a_{13} e,$$

where

$$a_{11} = \frac{\bar{Y}}{\bar{Y}_f} \qquad a_{12} = \frac{\bar{W}\bar{L}}{\bar{Y}_f \bar{P}_D},\ \qquad a_{13} = \frac{\bar{E}\bar{P}_R \bar{R}}{\bar{Y}_f \bar{P}_D};$$

(2.17) $l = a_{14}y = y;$

(2.18) $r = a_{14}y = y$

(i.e. $a_{14} = 1$);

(2.19) $p_D = a_{15}w + a_{16}p_R + a_{17}p_F + (a_{16} + a_{17})e,$

where

$$a_{15} = \frac{P_{D1}\bar{W}}{\bar{P}_D}, \qquad a_{16} = \frac{P_{D2}\bar{E}\bar{P}_R}{\bar{P}_D}, \qquad a_{17} = \frac{P_{D3}\bar{E}\bar{P}_F}{\bar{P}_D};$$

(2.20) $\dot{w} = a_{18}l,$

where

$$a_{18} = \bar{L}f_1.^{6}$$

We see that (2.12) - (2.20) are essentially the same as the log-linear 'detrended' specifications of (2.1) - (2.10). It should be noted that, in general, when deriving variational equations, non-linear equations are linearized about time paths. However, in this special case they are linearized at time points.

Now we obtain

$$\begin{aligned} y = {} & (-b_1a_{15} + b_1 + (b_2 + b_3)a_{15} - b_2 - b_4a_{15})w - \\ & - (b_1a_{16} + b_5 + (b_2 + b_3)a_{16} + b_4a_{16})p_R - \\ & - (-b_1a_{17} + (b_2 + b_3)a_{17} + b_4(1 - a_{17}))p_F + \\ & + (-b_1 - b_4(a_{16} + a_{17}) + b_4 - b_1(a_{16} + a_{17}))e + \\ & + (b_2 + b_3)(a_{16} + a_{17})e + (b_1 + b_6 - b_2 - b_5)y + \\ & + a_3a_9y^*, \end{aligned}$$

where

$$b_1 = a_1a_4a_{10}\,,\ b_2 = a_2a_6a_{12},\ b_3 = a_2a_6a_{13},$$

$$b_4 = a_1a_5 + a_2a_7 + a_3a_8\,,\ b_5 = a_2a_6a_{13}\,,\ b_6 = a_2a_6a_{11}.$$

Solving this for y yields

$$y = - c_1w + c_2p_F + c_3p_R + c_4e + c_5y^*, \tag{2.21}$$

where

$$c_1 = - (b_1 - b_2 - (b_1 - b_2 - b_3 + b_4)a_{15})/(1 - b_1 + b_2 + b_5 - b_6),$$

$$c_2 = (b_4 - (b_1 - b_2 - b_3 + b_4)a_{17})/(1 - b_1 + b_2 + b_5 - b_6),$$

$$c_3 = - (b_5 + (b_1 + b_2 + b_3 + b_4)a_{16})/1 - b_1 + b_2 + b_5 - b_6),$$

$$c_4 = \frac{b_4 - b_1 - (b_1 - b_2 - b_3 + b_4)(a_{16} + a_{17})}{(1 - b_1 + b_2 + b_5 - b_6)}$$

$$c_5 = a_3a_9/(1 - b_1 + b_2 + b_5 - b_6).$$

It can be seen that the signs of c_i $(i = 1,...,5)$ are ambiguous. For example, the short-run effect of a devaluation is negative (i.e. $c_4<0$). If the use of raw materials is important, if substitution effects are not strong and if a negative effect on households' incomes is not offset by a positive effect on firms' profits.

By (2.17) we can insert (2.21) into (2.20). We obtain

$$w(t) = w(0)e^{-a_{18}c_1t} + a_{18}c_2\int_0^t e^{-a_{18}c_1(t-u)}p_F(u)du + \tag{2.22}$$

$$+ a_{18}c_3\int_0^t e^{-a_{18}c_1(t-u)}p_R(u)du + a_{18}c_4\int_0^t e^{-a_{18}c_1(t-u)}e(u)du +$$

$$+ a_{18}c_5 \int_0^t e^{-a_{18}c_1(t-u)} y^*(u)du.$$

We assume that the system is stable i.e. that $c_1>0$.

Furthermore, we assume that $w(0) = 0$.

Using (2.22) we can evaluate the consequences of changes in P_R, P_F, E and Y^*. In the following chapter we show how one can use the variational equations derived in analysing the merits and drawbacks of the alternative exchange rate rules.

3. Exchange Rate Policy Rules

3.1. Fixed Exchange Rates and Foreign Price Distrubances

In this section we analyse how an upward shift in the foreign price level affects the economy under the fixed exchange rates rule.

We assume that the foreign price level increases by 1 % i.e. we assume that $p_F(t) = 1$ for $t \geq 0$ and study by deriving policy multipliers what effects it has on domestic prices, wages and production (employment).

When it is assumed that $p_F(t) = 1$, $p_R(t) = e(t) = y^*(t) = 0$ it follows from (2.22) that

$$(3.1) \qquad w(t) = \frac{c_2}{c_1}\,(1 - e^{-a_{18}c_1 t}).$$

This is the dynamic multiplier for nominal wages under the assumption that foreign price level increases by 1 %.

The impact effect is

$$\lim_{t \to 0} w(t) = 0$$

and the long-run effect

$$\lim_{t \to \infty} w(t) = c_2/c_1.$$

By (3.1) the convergence is monotonic. Under the assumption that c_1 and c_2 are positive, nominal wages will increase and the amount of the increase depends on the ratio c_2/c_1.

Correspondingly, by substituting (2.22) into (2.19) we have

$$(3.2) \qquad p_D(t) = \frac{a_{15}c_2}{c_1} + a_{17} - \frac{a_{15}c_2}{c_1} e^{-a_{18}c_1 t} .$$

Now the impact effect is

$$\lim_{t \to 0} p_D(t) = a_{17}$$

and the long-run effect

$$\lim_{t \to \infty} p_D(t) = a_{15}c_2/c_1 + a_{17}$$

respectively.

Using (3.1) and (3.2) we can compute the dynamic multiplier for real wages:

$$(3.3) \qquad w(t) - p_D(t) = \frac{c_2(1 - a_{15})}{c_1} - a_{17} + \frac{c_2(1 - a_{15})}{c_1} e^{-a_{18}c_1 t}.$$

Now

$$\lim_{t \to 0} (w(t) - p_D(t)) = -a_{17},$$

which is negative and

$$\lim_{t \to \infty} (w(t) - p_D(t)) = \frac{c_2(1 - a_{15})}{c_1} - a_{17}.$$

which can be negative or positive.

Inserting (2.22) into (2.21), the dynamic multiplier for the real output (and by (2.17) for employment) can be written in the form

$$(3.4) \qquad y(t) = c_2 e^{-a_{18}c_1 t} .$$

The impact effect is

$$\lim_{t \to 0} y(t) = c_2$$

and the long-run effect

$$\lim_{t \to \infty} y(t) = 0.$$

3.2. The Inflation Norm and Foreign Price Disturbances

We now assume that the authorities follow the inflation norm. We assume that if the price level of foreign goods rises by 1 % the authorities undertake a revaluation of equal magnitude i.e. we assume that $p_F(t) + e(t) = 0$. This is, of course, only one possible interpretation of the inflation norm.

Under the assumption that $p_F(t) = 1$, $p_R(t) = y^*(t) = 0$, and $e(t) = -1$ for all $t \geq 0$, we get the following dynamic multipliers:

By (2.22) the dynamic multiplier for nominal wages is

$$(3.5) \qquad w(t) = \frac{c_2 - c_4}{c_1} - \frac{c_2 - c_4}{c_1} e^{-a_{18} c_1 t}.$$

The impact effect is

$$\lim_{t \to 0} w(t) = 0$$

and the long-run effect

$$\lim_{t \to \infty} w(t) = \frac{c_2 - c_4}{c_1}.$$

We see that, if c_4 is negative nominal wages will now be higher in the long run than in the case of fixed exchange rates.

By (2.22) and (2.19) the dynamic multiplier for domestic prices is

$$(3.6) \qquad p_D(t) = \frac{a_{15}(c_2 - c_4)}{c_1} - a_{16} - \frac{a_{15}(c_2 - c_4)}{c_1} e^{-a_{18}c_1 t}.$$

Hence

$$\lim_{t \to 0} p_D(t) = -a_{16}$$

and

$$\lim_{t \to \infty} p_D(t) = \frac{a_{15}(c_2 - c_4)}{c_1} - a_{16}.$$

It is obvious that the price level will be lower both in the short run and in the long run when the inflation norm is followed instead of the fixed exchange rate norm.

By (3.5) and (3.6) the dynamic multiplier for real wages is

$$(3.7) \qquad w(t) - p_D(t) = (1 - a_{15}) \frac{c_2 - c_4}{c_1} + a_{16} -$$

$$- (1 - a_{15}) \frac{c_2 - c_4}{c_1} e^{-a_{18}c_1 t}.$$

Therefore in the short run

$$\lim_{t \to 0} (w(t) - p_D(t)) = a_{16}$$

and in the long run

$$\lim_{t \to \infty} (w(t) - p_D(t)) = (1 - a_{15}) \frac{c_2 - c_4}{c_1} + a_{16}.$$

We see that real wages are now higher than in the case of the fixed exchange rate rule in the short run and highly probably in the long run as well.

When calculating the dynamic multiplier for the domestic good we obtain

$$(3.8) \qquad y(t) = (c_2 - c_4)e^{-a_{18}c_1 t} .$$

Now we have:

$$\lim_{t \to 0} y(t) = c_2 - c_4$$

and

$$\lim_{t \to \infty} y(t) = 0$$

i.e. if c_4 is negative and the authorities follow the inflation norm the development of output (and employment) will be more favourable than in the case of the fixed exchange rates in the short run and in the intermediate run.

3.3. The Competitiveness Norm and Foreign Price Disturbances

Analytically the most interesting alternative is the competitiveness norm. We shall see that neutralizing a once-for-all change in the foreign price level will lead to continuing changes in the exchange rate if the authorities follow the competitiveness norm.

We assume that $p_F(t)$ is known and $p_R(t) = y^*(t) = 0$ for all $t \geq 0$. In addition we assume that the authorities follow the competitiveness norm i.e. we assume that

$$(3.9) \qquad p_F(t) + e(t) = p_D(t)$$

for all $t \geq 0$.

The problem we now pose is as follows: What is the reaction of the authorities if $p_F(t)$ is known and the authorities follow the rule (3.9)? When solving this problem we have to solve (3.9) with $e(t)$ as an unknown function.

In order to get the solution we have to solve an integral equation. It can be shown (for details, see Appendix A) that the solution for $e(t)$ can be expressed in the form

$$(3.10)\qquad e(t) = \frac{d_2}{d_4}\, p_F(t) + \frac{d_2}{d_4}\left(\frac{d_1}{d_2} + \frac{d_3}{d_4}\right) \int_0^t e^{-\left(a_{18}c_1 - \frac{d_3}{d_4}\right)(t-u)}\, p_F(u)\,du,$$

where

$$d_1 = a_{15}a_{18}c_2, \qquad d_2 = a_{17} - 1,$$

$$d_3 = a_{15}a_{18}c_4, \qquad d_4 = 1 - a_{16} - a_{17}.$$

This is the reaction function corresponding to the competitiveness norm under the assumption that the only disturbances are changes in the foreign price level.[7]

In order to get a concrete example we assume that $p_F(t) = 1$ for all $t \geq 0$ (a once-for-all change in the foreign price level) and calculate the reaction compatible with (3.10). We obtain

$$(3.11)\qquad e(t) = \frac{d_2}{d_4}\left(1 + \frac{d_1d_4 + d_2d_3}{d_2(a_{18}d_1c_1 + d_3)} - \frac{d_1d_4 + d_2d_3}{d_2(d_4a_{18}c_1 + d_3)}\, e^{-(a_{18}c_1 - d_3/d_4)t}\right)$$

Therefore

$$\lim_{t\to 0} e(t) = \frac{a_{17} - 1}{1 - a_{16} - a_{17}} < -1$$

(it is obvious that $0 < 1 - a_{16} - a_{17} < 1$)

and (under the assumption that $a_{18}c_1 - d_3/d_4 > 0$)

$$\lim_{t\to\infty} e(t) = \frac{(a_{17} - 1) + a_{15}c_2/c_1}{(1 - a_{16} - a_{17}) - a_{16}c_4/c_1}$$

Hence immediately after the price increase the authorities have to revalue the exchange rate by more than 1 %. Thereafter they have to go on changing monotonically the exchange rate. It depends on the structure of the economy whether they have to devalue or revalue. We see that if c_1, c_2 are positive and c_4 is negative then the authorities have to begin to devalue. The sign of e(t) in the long run is ambiguous, but it is obvious that e(t) is negative also in the long run.

For the authorities it is important to realize that the neutralization of a once-and-for-all increase in the foreign price level calls for continuing changes in the exchange rate if the competitiveness norm is followed.

The result obtained also shows why it is difficult by simulations based on the use of an econometric model to produce exchange rate policies which are compatible with the competitiveness norm. E.g. Halttunen (1981) and Halttunen-Korkman (1983) don't provide simulations of that kind. Nor does Korkman derive such policies in Korkman (1980).

By using (3.11) and $p_F(t) = 1$ we can calculate dynamic multipliers similar to those above. The expressions become, however, more complex: e.g. it can be shown (by inserting (3.11) and $p_F(t) = 1$ into (2.22)) that the dynamic multiplier for nominal wages is of the form

$$(3.12) \qquad w(t) = g_1 + g_2 e^{-a_{18}c_1 t} + g_3 e^{-(a_{18}c_1 - d_3/d_4)t},$$

where g_1, g_2 and g_3 are non-negative or negative constants. We see that the convergence is not necessarily monotonic: the multiplier can have a peak.

It can be shown that the impact effect is

$$\lim_{t \to 0} w(t) = 0$$

and the long-run effect

$$\lim_{t \to \infty} w(t) = \frac{c_2}{c_1} - \frac{c_4}{c_1} \left(\frac{1 - a_{17} - a_{15}c_2/c_1}{1 - a_{16} - a_{17} - a_{15}c_4/c_1} \right).$$

It depends on the structure of the economy whether nominal wages are now at a higher level in the long run than in the case of the inflation norm.

The dynamic multiplier for the domestic price level is of the form analogous to (3.12). It can be seen that

$$\lim_{t \to 0} p_D(t) = - \frac{a_{16}}{1 - a_{16} - a_{17}}$$

and

$$\lim_{t \to \infty} p_D(t) = \frac{a_{15}c_2}{c_1} + a_{17} - (a_{16} + a_{17}) \left(\frac{1 - a_{17} - a_{15}c_2/c_1}{1 - a_{16} - a_{17} - a_{15}c_4/c_1} \right)$$

Neither in this case is the convergence necessarily monotonic.

Combining these results we obtain the impact multiplier and the long-run multiplier for real wages:

$$\lim_{t \to 0} (w(t) - p_D(t)) = \frac{a_{16}}{1 - a_{16} - a_{17}}$$

and

$$\lim_{t\to\infty} (w(t) - p_D(t)) = (1 - a_{15})\frac{c_2}{c_1} - a_{17} - (c_4/c_1 - a_{16} - a_{17}) \frac{1 - a_{17} - a_{15}c_2/c_1}{1 - a_{16} - a_{17} - a_{15}c_4/c_1} .$$

We see that, if the authorities follow the competitiveness norm instead of the inflation norm, real wages will initially be higher.

However, this is not necessarily true in the long run. It must also be remembered that the convergence can be non-monotonic if the competitiveness norm is followed.

In a similar way, we can derive the dynamic multiplier for the real output. It can be expressed in the form analogous to (3.12). Now

$$\lim_{t\to 0} y(t) = c_2 - c_4\left(\frac{1 - a_{17}}{1 - a_{16} - a_{17}}\right)$$

and

$$\lim_{t\to\infty} y(t) = 0$$

We see that the sign of c_4 plays an important role when comparisons between the alternative exchange rate policy rules are made.

In this chapter, we have illustrated how variational analysis can be used in analysing the merits and drawbacks of the three exchange rate policy rules introduced by Korkman. We have seen that, if one wants to conduct an theoretical analysis similar to that based on simulation experiments which are performed by using an econometric model, variational analysis is a proper means for that purpose.

Notes

1 This equation has been derived as follows:

$$y = \frac{Y - \bar{Y}}{\bar{Y}} = \frac{\bar{C}^D}{\bar{Y}} \frac{C^D - \bar{C}^D}{\bar{C}^D} + \frac{\bar{I}^D}{\bar{Y}} \frac{I^D - \bar{I}^D}{\bar{I}^D} + \frac{\bar{X}}{\bar{Y}} \frac{X - \bar{X}}{\bar{X}} + \frac{\bar{G}}{\bar{Y}} \frac{G - \bar{G}}{\bar{G}}$$

In addition, we have assumed that $\delta G = 0$. We assume that all a - parameters in this paper are positive.

2 We use the notation f_i $(i = 1,...,n)$ to denote the partial derivative $\partial f/\partial x_i$ of $f(x_1,...,x_i,...x_n)$. In this paper, partial derivatives are evaluated at the reference state. In order to illustrate how (2.12) has been obtained we transform the function $y = f(x)$, which is assumed to be differentiable, into a variational form: linearizing at $X = \bar{X}$ yields $Y - \bar{Y} = f'(\bar{X})(X - \bar{X})$. This gives

$$y = \frac{Y - \bar{Y}}{\bar{Y}} = \frac{\bar{X} f'(\bar{X})}{\bar{Y}} \frac{(X - \bar{X})}{\bar{X}} = a\, x,$$

where $a = \dfrac{\bar{X} f'(\bar{X})}{\bar{Y}}$.

3 We assume that $\delta r = 0$.

4 We assume that $\delta J = \delta T_h = 0$.

5 We assume that $\delta T_f = 0$.

6 In this case $\dot{w} = \dot{W}/W$.

7 It is not difficult to derive an expression similar to (3.10) which allows, in addition to changes in the foreign price level, other disturbances to exist as well.

Appendix A

In this appendix, we solve (3.9) with e(t) as an unknown and $p_F(t)$ as a known function. When solving the equation we use Laplace transforms. (A very short introduction into the theory of Laplace transforms is presented in Appendix B.) We denote the Laplace transform of f(t) by L(f(t)) or by F(s). Assuming that Laplace transforms exist for functions $p_F(t)$, e(t) and $p_d(t)$, (3.9) can be transformed (by property 3 in Appendix B) into the form

$$(1) \qquad P_F(s) + E(s) = P_D(s).$$

Assuming that $p_R(t) = y^*(t) = 0$ (i.e. we only are interested in changes in the foreign price level), (2.22) can be transformed (by properties 2, 3 and 4) into the form

$$(2) \qquad W(s) = \frac{a_{18}c_2}{s + a_{18}c_1} P_F(s) + \frac{a_{18}c_4}{s + a_{18}c_1} E(s).$$

Taking the Laplace transform from (2.19) and inserting (2) into it we obtain

$$(3) \qquad P_D(s) = \left(\frac{a_{15}a_{18}c_2}{s + a_{18}c_1} + a_{17}\right)P_F(s) + \left(a_{16} + a_{17} + \frac{a_{15}a_{18}c_4}{s + a_{18}c_1}\right)E(s).$$

Substituting (3) into (1) yields

$$(4) \qquad P_F(s) + E(s) = \left(\frac{a_{15}a_{18}c_2}{s + a_{18}c_1} + a_{17}\right)P_F(s) + \left(a_{16} + a_{17} + \frac{a_{15}a_{18}c_4}{s + a_{18}c_1}\right)E(s).$$

This equation defines an integral equation expressed in the Laplace transform form with E(s) as an unknown function and $P_F(s)$ as a known function. (Using property 4 it is easy to see that (4) really defines an integral equation.) Solving (4) for E(s) gives

$$E(s) = \frac{(a_{17} - 1)(s + a_{18}c_1) + a_{15}a_{18}c_2}{(1 - a_{16} - a_{17})(s + a_{18}c_1) - a_{15}a_{18}c_4} P_F(s)$$

$$= \frac{a_{17} - 1}{1 - a_{16} - a_{17}} \left(P_F(s) + \frac{a_{15}a_{18}c_2}{a_{17} - 1} + \frac{a_{15}a_{18}c_4}{1 - a_{16} - a_{17}}\right) \frac{P_F(s)}{s + a_{18}c_1} - \frac{a_{15}a_{18}c_4}{1 - a_{16} - a_{17}}$$

Inverting this we get by properties 1, 2, 3 and 4 equation (3.10). Thus by using Laplace transforms, we were able to solve an integral equation by solving an algebraic equation (equation (4)).

Appendix B Laplace Transforms[1]

The basic idea in the use of Laplace transforms is that one can solve e.g. linear differential and integral equations by solving algebraic equations.

Definition: Suppose that $f(t)$ is defined for all $t \geq 0$. The Laplace transform of f, which is denoted by $F(s)$ or $L(f(t))$ is defined by the equation

$$L(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)dt,$$

whenever this integral converges.

In general s can be complex, but we consider only real values of s.

Property 1. Let $f(t) = a$, $t \geq 0$. Then

$$L(a) = a/s, \quad s>0.$$

Property 2. Let $F(s) = L(f(t))$. Then

$$L(e^{at}f(t)) = F(s-a).$$

Property 3 (linearity). Assume that Laplace transforms for functions f_1 and f_2 exist. Then

$$L(c_1f_1 + c_2f_2) = c_1L(f_1) + c_2L(f_2),$$

where c_1 and c_2 are arbitrary constants.

Property 4 (Laplace transforms for convolution integrals). Assume $F(s) = L(f(t))$ and $G(s) = L(g(t))$ both exist for $s \geq a>0$. Then

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[1] This appendix is based on Boyce - DiPrima (1965).

$$H(s) = F(s)G(S) = L(h(t)), \quad s>a,$$

where

$$h(t) = \int_0^t f(t-u)g(u)du = \int_0^t f(u)g(t-u)du$$

is the convolution of f and g.

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