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THE INTERDEPENDENCE OF THE EXCHANGE RATE POLICIES OF TWO SMALL COMPETING COUNTRIES

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Essay written for Professors Willem Buiter and Marcus Miller at the VI Yrjö Jahnsson Graduate Course on Macroeconomics

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THE INTERDEPENDENCE OF THE EXCHANGE RATE POLICIES OF TWO SMALL COMPETING COUNTRIES

1. Introduction

In this study, we analyse the interdependence of the exchange rate policies of two small countries competing with each other. The concrete cause for writing this essay is, of course, the chain of devaluations undertaken by Finland and Sweden last autumn. Our main objective is to introduce a framework by which an interdependence of this kind can be studied at an abstract level.

The discussion of the exchange rate policy rules has been one source of ideas. The main contributions to this debate have been given by Korkman, who provides a theoretical discussion of three alternative rules for exchange rate policy: the fixed exchange rate rule, the inflation norm, and the competitiveness norm. (See Korkman, 1980, pp. 79-86).[1] In his study, Korkman discusses the usefulness of these rules in neutralizing various exogenous shocks.

We also introduce some norms analogous to those of Korkman. However, the target of the norms presented in this essay is the neutralization of one specific exogenous disturbance not analysed by Korkman: the exchange rate measures taken by the competitor of our home country. One additional remark is worth noticing. We shall not suggest that one of the norms introduced in this paper should be followed by the authorities. By the norms, we try to describe one aspect of the real decision making process.

When conducting the analysis, we use Aoki´s method for analysing dynamic interdependent systems. (See Aoki, 1981). By this method, we can study, among other things, one important but easily neglected aspect associated with the interdependence of the exchange rate policies: how do structural differences between the countries affect the nature of interdependence? This is one of the principal questions we deal with in this paper.

We performe all considerations by using a very simple three-country model which is set up in chapter 2. In chapter 3 we carry out policy evaluations of some exchange rate policy measures and in chapter 4 we study the problem of the interdependence of the exchange rate policies more thoroughly. Some concluding comments are presented in chapter 5.

2. The Model

2.1. The Model in the Level Form

In this chapter, we construct a very simple three-country model by which we can illustrate the basic ideas.

Two of the countries, A (home country) and B (neighbouring country, the competitor of A), are small and one, C (the rest of the world), is large.

The countries produce nonidentical goods, which can be used as substitutes. To keep the model as simple as possible, only goods markets are considered.

The authorities pursuing economic policy in countries A and B have one instrument, the exchange rates, at their disposal. We assume that country C doesn´t use this instrument (i.e. fixed exchange rates prevail in the rest of the world).

The dynamics is brought into the model by the inflation dynamics of countries A and B. The price level in country C is assumed to be fixed.

The level form of the model is as follows:

(2.1) $$Y^A = Y^A(P^B E_{AB}/P^A,\ \bar{P}^C E_{AC}/P^A, Y^B, \bar{Y}^C),$$

(2.2) $$Y^B = Y^B(P^A E_{BA}/P^B,\ \bar{P}^C E_{BC}/P^B, Y^A, \bar{Y}^C),$$

(2.3) $$\dot{P}^A/P^A = f^A(Y^A),$$

(2.4) $\dot{P}^B/P^B = f^B(Y^B)$,

where

Y^i = the real output produced by country i, i = A,B,
$\bar{Y}^C$ = the real output produced by country C (fixed),
P^i = the price level of country i, i = A,B,
$\bar{P}^C$ = the price level of country C (fixed),

E_{ij} = the price of country j currency in terms of the currency of country i, i =A,B, j = A,B,C, $i \neq j$,
$\dot{P}^i$ = dP^i/dt, i = A,B.

We assume that Y^A, Y^B, f^A and f^B have continuous partial derivatives. As can be seen, the model is a very simple demand oriented Keynesian model. Equations (2.1) and (2.2) describe equilibrium conditions in the goods markets in countries A and B respectively. The relative prices play a major role in the output determination. The Phillips curves (2.3) and (2.4) govern the dynamic behaviour of the model.

However, we don´t use this form of the model when conducting the analysis. In order to be able to evaluate dynamic effects of the policy instruments, the variational form of the model is used. (For variational analysis see Aoki, 1981). Besides impact effects and long-run effects of policy instruments also inter-run effects can be analysed when variational equations are used.

2.2. The Model in the Variational Form

When forming the variational equations of (2.1) - (2.4) we need a reference path or point against which to compare. We assume that the model, which can be non-linear, has a unique long-run equilibrium state $\bar{Y}^A, \bar{Y}^B, \bar{P}^A$ and $\bar{P}^B$. This is the reference state of the economy against which alternate time paths of the economy resulting from changes in the policy instruments are compared in our model.

Using the notation $x = (X - \bar{X})/\bar{X} = \delta X/\bar{X}$, where $\bar{X}$ is the long-run equilibrium value of X (an endogenous variable) or the value of an exogenous variable associated with the reference state, and forming the variational equations of (2.1) - (2.4) we obtain (for details, see Aoki, 1981, pp. 15-20):

(2.5) $$y^A = a_1(p^B + e'_{AB} - p^A) + a_2(e_{AC} - p^A) + a_3 y^B,$$

(2.6) $$y^B = a_1^*(p^A + e'_{BA} - p^B) + a_2^*(e_{BC} - p^B) + a_3^* y^A,$$

(2.7) $$\dot{p}^A = b_1 y^A,$$

(2.8) $$\dot{p}^B = b_1^* y^B,$$

where

$$a_1 = \frac{\partial Y^A}{\partial(P^B E_{AB}/P^A)} \frac{\bar{E}_{AB}\bar{P}^B}{\bar{P}^A\bar{Y}^A} \qquad \text{etc,}^2$$

We assume that a_i^*, a_i (i = 1,2,3), b_1 and b_1^* are positive.

In (2.5) e_{AC} is a proportional change in E_{AC} caused by country A, and in (2.6) e_{BC} is a proportional change in E_{BC} caused by country B respectively. Furthermore, because both countries A and B can change E_{AB} and E_{BA} ($E_{AB} = 1/E_{BA}$), e'_{AB} (a proportional change in E_{AB}) is defined as

(2.9) $\qquad e'_{AB} = e_{AB} - e_{BA} = -e'_{BA}$,

where e_{AB} is a proportional change in E_{AB} (E_{BA}) caused by country A and e_{BA} is a proportional change in E_{AB} (E_{BA}) caused by country B. Hence e'_{AB} is the net change resulting from the changes in the two policy instruments.

In addition, we have

(2.10) $\qquad e_{AB} = e_{AC}$ and $e_{BA} = e_{BC}$.

It can be seen that the variational equations (2.5)-(2.8) of the non-linear model (2.1)-(2.4) are essentially the same as the log-linear (detrended) specifications of these equations. (See Aoki, 1981, pp. 15-20).

Using matrix representation and (2.9)-(2.10), (2.5)-(2.8) can be expressed in the form:

$$(2.11)\quad \begin{pmatrix} y^A \\ y^B \end{pmatrix} = \begin{pmatrix} -(a_1 + a_2) & a_1 \\ a_1^* & -(a_1^* + a_2^*) \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} +$$

$$\begin{pmatrix} a_1 + a_2 & -a_1 \\ -a_1^* & a_1^* + a_2^* \end{pmatrix} \begin{pmatrix} e_{AC} \\ e_{BC} \end{pmatrix} + \begin{pmatrix} 0 & a_3 \\ a_3^* & 0 \end{pmatrix} \begin{pmatrix} y^A \\ y^B \end{pmatrix}$$

$$(2.12)\quad \begin{pmatrix} \dot{p}^A \\ \dot{p}^B \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ 0 & b_1^* \end{pmatrix} \begin{pmatrix} y^A \\ y^B \end{pmatrix} .$$

This is the form of the model we use in the subsequent analysis.

3. Policy Evaluation of Some Exchange Rate Policy Measures

3.1. Structural Perturbation Analysis and Policy Evaluation

One of the objectives of Aoki´s book is to provide a general procedure for evaluating dynamic effects of national economic policies. Now we use Aoki´s approach when studying the interdependence of the exchange rate policies of countries A and B. Particularly, we are interested in how the structural differences between these two countries affect the nature of the interdependence. Therefore structural perturbation analysis presented in Aoki´s book is used.

The starting point of this approach is to assume that the countries are identical. This means that the structures and characteristics of these countries are similar. The policy evaluations are first conducted under this assumption. Then a sensitivity analysis is performed under the assumption that the countries differ in some respect. (For more details, see Aoki, 1981, ch. 5 and 13).

3.1.1. Identical Countries

We now assume that countries A and B are identical i.e. we assume that $a_i = a_i^*$ (i = 1,2,3) and $b_1 = b_1^*$.

Using a_i (i = 1,2,3) and b_1 to denote the common parameters, (2.11) and (2.12) become

(3.1) $$\begin{pmatrix} y^A \\ y^B \end{pmatrix} = \begin{pmatrix} -(a_1 + a_2) & a_1 \\ a_1 & -(a_1 + a_2) \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} + \begin{pmatrix} a_1 + a_2 & -a_1 \\ -a_1 & a_1 + a_2 \end{pmatrix} \begin{pmatrix} e_{AC} \\ e_{BC} \end{pmatrix} + \begin{pmatrix} 0 & a_3 \\ a_3 & 0 \end{pmatrix} \begin{pmatrix} y^A \\ y^B \end{pmatrix},$$

(3.2) $$\begin{pmatrix} \dot{p}^A \\ \dot{p}^B \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ 0 & b_1 \end{pmatrix} \begin{pmatrix} y^A \\ y^B \end{pmatrix}$$

We call (3.1)-(3.2) the benchmark model.

The representation of the dynamics of the model can be simplified by transforming (3.1) and (3.2) to forms equivalent to them. The model is described by using the averages and differences of the variables.

We define the average and difference of x^A and x^B as follows:

$$x_a = (x^A + x^B)/2 = \tfrac{1}{2}(1,1)\begin{pmatrix} x^A \\ x^B \end{pmatrix},$$

(3.3) $$x_d = (x^A - x^B)/2 = \tfrac{1}{2}(1,-1)\begin{pmatrix} x^A \\ x^B \end{pmatrix}$$

Using the matrix

$$E = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

we get

$$(3.4)\qquad \begin{pmatrix} x_a \\ x_d \end{pmatrix} = E \begin{pmatrix} x^A \\ x^B \end{pmatrix} .$$

Furthermore, we see that

$$E^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = 2E.$$

Thus

$$(3.5)\qquad \begin{pmatrix} x^A \\ x^B \end{pmatrix} = E^{-1} \begin{pmatrix} x_a \\ x_d \end{pmatrix} = 2E \begin{pmatrix} x_a \\ x_d \end{pmatrix} .$$

By (3.4) and (3.5) the model can be transformed into the following form:

$$(3.6)\qquad \begin{pmatrix} y_a \\ y_d \end{pmatrix} = E \begin{pmatrix} y^A \\ y^B \end{pmatrix} = E \begin{pmatrix} -(a_1 + a_2) & a_1 \\ a_1 & -(a_1+a_2) \end{pmatrix} 2E \begin{pmatrix} p_a \\ p_d \end{pmatrix}$$

$$+ E \begin{pmatrix} a_1 + a_2 & -a_1 \\ -a_1 & a_1 + a_2 \end{pmatrix} 2E \begin{pmatrix} e_a \\ e_d \end{pmatrix} + E \begin{pmatrix} 0 & a_3 \\ a_3 & 0 \end{pmatrix} 2E \begin{pmatrix} y_a \\ y_d \end{pmatrix}$$

$$= \begin{pmatrix} -a_2 & 0 \\ 0 & -2a_1-a_2 \end{pmatrix} \begin{pmatrix} p_a \\ p_d \end{pmatrix} + \begin{pmatrix} a_2 & 0 \\ 0 & 2a_1+a_2 \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix}$$

$$+ \begin{pmatrix} a_3 & 0 \\ 0 & -a_3 \end{pmatrix} \begin{pmatrix} y_a \\ y_d \end{pmatrix}$$

where $e_a = (e_{AC} + e_{BC})/2$ and $e_d = (e_{AC} - e_{BC})/2$;

$$(3.7)\qquad \begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = E \begin{pmatrix} \dot{p}^A \\ \dot{p}^B \end{pmatrix} = E \begin{pmatrix} b_1 & 0 \\ 0 & b_1 \end{pmatrix} 2E \begin{pmatrix} y_a \\ y_d \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ 0 & b_1 \end{pmatrix} \begin{pmatrix} y_a \\ y_d \end{pmatrix}.$$

Hence

$$(3.8)\qquad \begin{pmatrix} y_a \\ y_d \end{pmatrix} = \begin{pmatrix} -c_1 & 0 \\ 0 & -c_2 \end{pmatrix}\begin{pmatrix} p_a \\ p_d \end{pmatrix} + \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}\begin{pmatrix} e_a \\ e_d \end{pmatrix}$$

where $c_1 = (1-a_3)^{-1}a_2$ and $c_2 = (1+a_3)^{-1}(2a_1 + a_2)$. Inserting (3.8) into (3.7), we obtain

$$(3.9)\qquad \begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} -b_1c_1 & 0 \\ 0 & -b_1c_2 \end{pmatrix}\begin{pmatrix} p_a \\ p_d \end{pmatrix} + \begin{pmatrix} b_1c_1 & 0 \\ 0 & b_1c_2 \end{pmatrix}\begin{pmatrix} e_a \\ e_d \end{pmatrix}.$$

From (3.8) and (3.9), we see that if countries A and B have the same characteristics, the equations governing the behaviour of the average variables are completely separated from those governing the behaviour of the difference variables. This property turns out to be very useful. The solution of (3.9) is:

$$p_a(t) = e^{-b_1c_1t}\, p_a(0) + b_1c_1 \int_0^t e^{-b_1c_1(t-u)}\, e_a(u)du$$

and

$$p_d(t) = e^{-b_1c_2t} p_d(0) + b_1c_2 \int_0^t e^{-b_1c_2(t-u)} e_d(u)du.$$

Throughout this study, we assume that the initial conditions are equal to zero. Therefore

(3.10) $$p_a(t) = b_1c_1 \int_0^t e^{-b_1c_1(t-u)} e_a(u)du$$

(3.11) $$p_d(t) = b_1c_2 \int_0^t e^{-b_1c_2(t-u)} e_d(u)du \quad .$$

The average dynamics is stable if $b_1c_1>0$ i.e. if $0<a_3<1$. Since b_1c_2 is positive by the assumption made earlier, (3.11) is stable.

Substituting (3.10) and (3.11) into (3.8), we have

(3.12) $$y_a(t) = -b_1c_1^2 \int_0^t e^{-b_1c_1(t-u)} e_a(u)du + c_1e_a(t)$$

and

(3.13) $$y_d(t) = -b_1c_2^2 \int_0^t e^{-b_1c_2(t-u)} e_d(u)du + c_2e_d(t)$$

Now, (3.10)-(3.13) enable us to make some preliminary remarks about the consequences of the exchange rate policies pursued by countries A and B:

If the exchange rate policies are coordinated i.e. if $e_d = 0$, then by (3.11) and (3.13) $p_d(t) = y_d(t) = 0$. The behaviour of the two countries is identical.

It is also obvious (we prove this later) that in order to keep the countries on the same output or price path, coordinated exchange rate policies should be pursued. In other words, if country B devaluates, country A also has to undertake (immediately) a devaluation of equal magnitude if it is to stay on the same output and price path as country B.

In order to analyse the consequences of a non-coordinated policy, we assume that country B undertakes a once-and-for-all devaluation of one percentage and calculate dynamic multipliers for $p_a(t)$, $p_d(t)$, $y_a(t)$, and $y_d(t)$,

Now

$$e_a = (0 + 1)/2 = \tfrac{1}{2} \text{ and } e_d = (0-1)/2 = -\tfrac{1}{2}.$$

Thus by (3.11)

$$p_d(t) = -\tfrac{1}{2}\left(1 - e^{-b_1 c_2 t}\right) \tag{3.14}$$

and by (3.13)

$$y_d(t) = \frac{c_2}{2} e^{-b_1 c_2 t} . \tag{3.15}$$

The impact multipliers are

$$\lim_{t \to 0} p_d(t) = 0 , \qquad \lim_{t \to 0} y_d(t) = -c_2/2 , \tag{3.16}$$

and the long-run multipliers respectively

$$\lim_{t \to \infty} p_d(t) = -\tfrac{1}{2} , \qquad \lim_{t \to \infty} y_d(t) = 0 . \tag{3.17}$$

Hence immediately after the devaluation activity becomes higher in country B and it remains at a higher level also in the intermediate run. However, the difference between the activity levels decreases monotonically, and in the long run it is equal to zero. On the other hand, immediately after the devaluation the prices remain the same but thereafter prices rise faster in country B.[3]

Accordingly, the dynamic multipliers for p_a and y_a are

$$(3.18) \qquad p_a(t) = \tfrac{1}{2}\,(\,1 - e^{-b_1 c_1 t}\,),$$

$$(3.19) \qquad y_a(t) = \frac{c_1}{2}\,e^{-b_1 c_1 t}\,.$$

In the long run, the average price level will be higher. Instead, the average level of activity remains at the same level.

The main result we derived in this section was not very surprising: If countries A and B are identical, they will behave in a similar way, if coordinated exchange rate policy is pursued. On the other hand, if the countries are to behave in a similar way, coordinated exchange rate policy should be pursued.

In the subsequent sections, we study how this result must be modified, if the countries have not the same characteristics.

3.1.2. Non-identical Countries

We drop now the assumption of identical countries and study how the conclusions made in the previous section are changed.

We assume that countries A and B differ in one respect: we assume that $b_1 \neq b_1^*$, $b_1 > b_1^*$ i.e. inflation in country A is more vulnerable to changes in activity than in country B.

In this case, (3.8) remains the same but (3.9) becomes different: transforming (2.12) we obtain:

$$(3.20) \quad \begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = E \begin{pmatrix} b_1 & 0 \\ 0 & b_1^* \end{pmatrix} 2E \begin{pmatrix} y_a \\ y_d \end{pmatrix} =$$

$$\begin{pmatrix} b_a & b_d \\ b_d & b_a \end{pmatrix} \begin{pmatrix} y_a \\ y_d \end{pmatrix} ,$$

where $b_a = (b_1 + b_1^*)/2$ and $b_d = (b_1 - b_1^*)/2$. Using (3.8), (3.20) can be written in the form:

$$(3.21) \quad \begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} -b_a c_1 & -b_d c_2 \\ -b_d c_1 & -b_a c_2 \end{pmatrix} \begin{pmatrix} p_a \\ p_d \end{pmatrix} +$$

$$\begin{pmatrix} b_a c_1 & b_d c_2 \\ b_d c_1 & b_a c_2 \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix} .$$

This equation differs from (3.9) in that there are interaction terms between the average and difference variables. For instance, the term $-b_d c_2$ implies the existence of a transmission path from the difference variables to the average variables.

It can be seen that, if $0<a_3<1$ and the other parameters are positive, (3.21) is stable.

We solve (3.21) by making use of the solution of the benchmark model. (For details, see Aoki, 1981, pp. 192-194). Substituting b_a for b_1 in (3.7) and distinquishing the solution of (3.9) from that of (3.21) by superscript zero, we obtain:

$$(3.22) \qquad p_a^o(t) = b_a c_1 \int_0^t e^{-b_a c_1 (t-u)} e_a(u)du,$$

$$(3.23) \qquad p_d^o(t) = b_a c_2 \int_0^t e^{-b_a c_2 (t-u)} e_d(u)du.$$

Next, we define a new set of variational variables by

$$(3.24) \qquad \delta p_a = p_a - p_a^o \quad \text{and} \quad \delta p_d = p_d - p_d^o$$

and derive the solutions for δp_a and δp_d. Thereafter we use these solutions and the solution of the benchmark model in deriving the solutions for p_a and p_d.

Now, using (3.21) and the benchmark model, we have

$$\begin{pmatrix} \delta\dot{p}_a \\ \delta\dot{p}_d \end{pmatrix} = \begin{pmatrix} \dot{p}_a - \dot{p}_a^o \\ \dot{p}_d - \dot{p}_d^o \end{pmatrix} = \begin{pmatrix} -b_a c_1 & -b_d c_2 \\ -b_d c_1 & -b_a c_2 \end{pmatrix} \begin{pmatrix} p_a \\ p_d \end{pmatrix} +$$

$$\begin{pmatrix} b_a c_1 & b_d c_2 \\ b_d c_1 & b_a c_2 \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix} - \begin{pmatrix} -b_a c_1 & 0 \\ 0 & -b_a c_2 \end{pmatrix} \begin{pmatrix} p_a^o \\ p_d^o \end{pmatrix} -$$

$$-\begin{pmatrix} b_a c_1 & 0 \\ 0 & b_a c_2 \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix} =$$

$$\begin{pmatrix} -b_a c_1 & -b_d c_2 \\ -b_d c_1 & -b_a c_2 \end{pmatrix} \begin{pmatrix} \delta p_a \\ \delta p_d \end{pmatrix} + \begin{pmatrix} 0 & -c_2 b_d \\ -c_1 b_d & 0 \end{pmatrix} \begin{pmatrix} p_a^o \\ p_d^o \end{pmatrix} +$$

$$\begin{pmatrix} 0 & b_d c_2 \\ b_d c_1 & 0 \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix}.$$

Ignoring $-b_d c_2 \delta p_d$ and $-b_d c_1 \delta p_a$ as being small, we can express δp_a and δp_d as follows:

(3.25) $$\delta \dot{p}_a = -b_a c_1 \delta p_a - b_d c_2 p_d^o + b_d c_2 e_d ,$$

(3.26) $$\delta \dot{p}_d = -b_a c_2 \delta p_d - b_d c_1 p_a^o + b_d c_1 e_a .$$

(See Aoki, 1981, p. 193). The solutions for (3.25) and (3.26) are (assuming that $\delta p_a(0) = \delta p_d(0) = 0$):

(3.27) $$\delta p_a(t) = -b_d c_2 \int_0^t e^{-b_a c_1 (t-u)} p_d^o(u) du + b_d c_2 \int_0^t e^{-b_a c_1 (t-u)} e_d(u) du,$$

(3.28) $$\delta p_d(t) = -b_d c_1 \int_0^t e^{-b_a c_2 (t-u)} p_a^o(u) du + b_d c_1 \int_0^t e^{-b_a c_2 (t-u)} e_a(u) du.$$

We could next substitute (3.23) into (3.27) and (3.22) into (3.28) in order to express the solutions by e_a and e_d only. Thereafter we could use the solutions for p_a^o and δp_a in deriving the solution for p_a, and the solutions for p_d^o and δp_d in deriving the solution for p_d. In fact we do that, but we don't use the time domain representations of the solutions. Instead, we use expressions based on the method of Laplace transforms.[4] (A very short introduction into the theory of Laplace transforms is presented in Appendix A).

We denote the Laplace transform of f(t) by L(f(t)) or by F(s). If not explicitly stated, we implicitly assume that Laplace transforms exist for the functions we use.

Next, we derive the solutions for p_a and p_d by using Laplace transforms. By properties 3 and 4 (in Appendix A), (3.25) and (3.26) can be transformed into the forms

$$s\delta P_a(s) = -b_a c_1 \delta P_a(s) - b_d c_2 P^o(s) + b_d c_2 E_d(s),$$

$$s\delta P_d(s) = -b_a c_2 \delta P_d(s) - b_d c_1 P_a^o(s) + b_d c_1 E_a(s).$$

Solving for $\delta P_a(s)$ and $\delta P_d(s)$ we obtain

$$\delta P_a(s) = -\frac{b_d c_2}{s+b_a c_1} P_d^o(s) + \frac{b_d c_2}{s+b_a c_1} E_d(s), \tag{3.29}$$

$$\delta P_d(s) = -\frac{b_d c_1}{s+b_a c_2} P_a^o(s) + \frac{b_d c_1}{s+b_a c_2} E_a(s). \tag{3.30}$$

By properties 1,2 and 5 we see that (3.29) and (3.30) are equivalent to (3.27) and (3.28) (i.e. by computing the inverse transforms $L^{-1}(\delta P_a(s))$ and $L^{-1}(\delta P_d(s))$ we get (3.27) and (3.28).) Since (3.22) and (3.23) are convolution integrals we have by properties 1,2 and 5

$$(3.31) \qquad P_a^o(s) = \frac{b_a c_1}{s+b_a c_1} E_a(s),$$

$$(3.32) \qquad P_d^o(s) = \frac{b_a c_2}{s+b_a c_2} E_d(s).$$

Now, substituting (3.31) into (3.30) and (3.32) into (3.29) and using (3.24) we can express the solutions for p_a in the Laplace transform form as follows:

$$(3.33) \qquad P_a(s) = P_a^o(s) + \delta P_a(s) = \frac{b_a c_1}{s+b_a c_1} E_a(s) +$$

$$+ \frac{b_d c_2^2}{c_2 - c_1} \left(\frac{1}{s+b_a c_2} - \frac{1}{s+b_a c_1}\right) E_d(s) +$$

$$+ \frac{b_d c_2}{s+b_a c_1} E_d(s),$$

$$(3.34) \qquad P_d(s) = P_d^o(s) + \delta P_d(s) = \frac{b_a c_2}{s+b_a c_2} E_d(s) +$$

$$+ \frac{b_d c_1^2}{c_2 - c_1} \left(\frac{1}{s+b_a c_2} - \frac{1}{s+b_a c_1}\right) E_a(s) +$$

$$+ \frac{b_d c_1}{s+b_a c_2} E_a(s).$$

From (3.33) and (3.34) we see that averages and differences are no longer completely separated. In order to illustrate how this property is reflected in the behaviour of countries A and B, we calculate dynamic multipliers for p_a, p_d, y_a and y_d.

By (3.8) and property 3

(3.35) $\quad Y_a(s) = -c_1 P_a(s) + c_1 E_a(s),$

(3.36) $\quad Y_d(s) = -c_2 P_d(s) + c_2 E_d(s).$

When calculating the dynamic multipliers, we use equations (3.32)-(3.36)

<u>Coordinated Devaluation, $b_1 \neq b_1^*$</u>

Assume that a coordinated once-and-for-all devaluation of 1 % is undertaken by countries A and B. This implies that e_d (and E_d) is zero and that $E_a(s)=1/s$ by property 1. Using (3.34) we obtain after some algebra

$$P_d(s) = \frac{b_d c_1}{b_a(c_2-c_1)} \left(\frac{1}{s+b_a c_1} - \frac{1}{s+b_a c_2}\right)$$

and by (3.36)

$$Y_d(s) = -\frac{b_d c_1 c_2}{b_a(c_2-c_1)} \left(\frac{1}{s+b_a c_1} - \frac{1}{s+b_a c_2}\right).$$

Taking the inverse transforms $L^{-1}(P_d(s))$ and $L^{-1}(Y_d(s))$ we have by properties 1,2 and 3

$$(3.37) \qquad p_d(t) = \frac{b_d c_1}{b_a(c_2-c_1)} \left(e^{-b_a c_1 t} - e^{-b_a c_2 t} \right),$$

$$(3.38) \qquad y_d(t) = \frac{b_d c_1 c_2}{b_a(c_2-c_1)} \left(e^{-b_a c_1 t} - e^{-b_a c_2 t} \right).$$

From (3.37) and (3.38) we see that

$$\lim_{t\to 0} p_d(t) = \lim_{t\to\infty} p_d(t) = 0$$

and

$$\lim_{t\to 0} y_d(t) = \lim_{t\to\infty} y_d(t) = 0.$$

Thus, the impact multipliers and the long-run multipliers are the same as in the case of identical countries. The structural difference between the countries does not give rise to a differing behaviour in the short run and in the long run. However, in the intermediate run it does: from (3.37) and (3.38) it can be seen that in the intermediate run $p_d(t)>0$ and $y_d(t)<0$. Country A, where inflation is more vulnerable to changes in activity, has a lower level of activity in the intermediate run due to a higher rate of inflation. We also see that the convergence is not necessarily monotonic: the multipliers can have peaks.

Accordingly, we can draw the following conclusion: if countries A and B are not identical, coordinated exchange rate policies dont´t ensure that they would behave in a similar way (for all $t\geq 0$).

By illustration, we also compute the dynamic multipliers for p_a and y_a under the assumption of a coordinated once-and-for-all devaluation. By (3.33) we have

$$P_a(s) = \frac{b_a c_1}{s+b_a c_1} \frac{1}{s} = \frac{1}{s} - \frac{1}{s+b_a c_1}$$

and therefore by (3.35)

$$Y_a(s) = - \frac{b_a c_1^2}{s+b_a c_1} \frac{1}{s} + \frac{c_1}{s} = \frac{c_1}{s+b_a c_1} .$$

Inverting these, $p_a(t)$ and $y_a(t)$ can be written as follows (by properties 1,2 and 3):

$$(3.39) \qquad p_a(t) = 1 - e^{-b_a c_1 t},$$

$$(3.40) \qquad y_a(t) = c_1 e^{-b_a c_1 t},$$

which are similar to (3.18) and (3.19).

Newt, we illustrate how the difference between the countries is reflected when uncoordinated policies are pursued.

<u>Unilateral Devaluation by Country B, $b_1 \neq b_1^*$</u>

We assume that a unilateral devaluation of 1 % is undertaken by country B and compute the dynamic multipliers.

Using (3.34) we obtain

$$P_d(s) = -\frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+b_a c_2}\right) +$$

$$\frac{b_d c_1}{2b_a(c_2-c_1)}\left(\frac{1}{s+b_a c_2} - \frac{1}{s+b_a c_1}\right).$$

Inverting this, $p_d(t)$ can be written in the form

$$p_d(t) = -\frac{1}{2}\left(1 - e^{-b_a c_2 t}\right) + \tag{3.41}$$

$$\frac{b_d c_1}{2b_a(c_2-c_1)}\left(e^{-b_a c_2 t} - e^{-b_a c_1 t}\right).$$

Similarly, by (3.36)

$$y_d(t) = -\left(\frac{c_2}{2} + \frac{b_d c_1 c_2}{2b_a(c_2-c_1)}\right)e^{-b_a c_2 t} + \tag{3.42}$$

$$\frac{b_d c_1 c_2}{2b_a(c_2-c_1)} e^{-b_a c_1 t}.$$

Hence,

$$\lim_{t\to 0} p_d(t) = 0, \quad \lim_{t\to\infty} p_d(t) = -\tfrac{1}{2},$$

and

$$\lim_{t\to 0} y_d(t) = -c_2/2, \quad \lim_{t\to\infty} y_d(t) = 0.$$

We see that the impact effects and the long-run effects are the same as in the case of identical countries. (See (3.16) and(3.17).) However, the intermediate run effects differ. E.g. unlike (3.15), (3.42) does not necessarily converge monotonically.

We have shown that, if countries A and B differ, the effects of the exchange rate measures taken are not the same as those in the case of identical countries. Particularly, if countries A and B are not identical, coordinated exchange rate policies don´t have identical effects on these countries.

We illustrate these results by numerical examples.

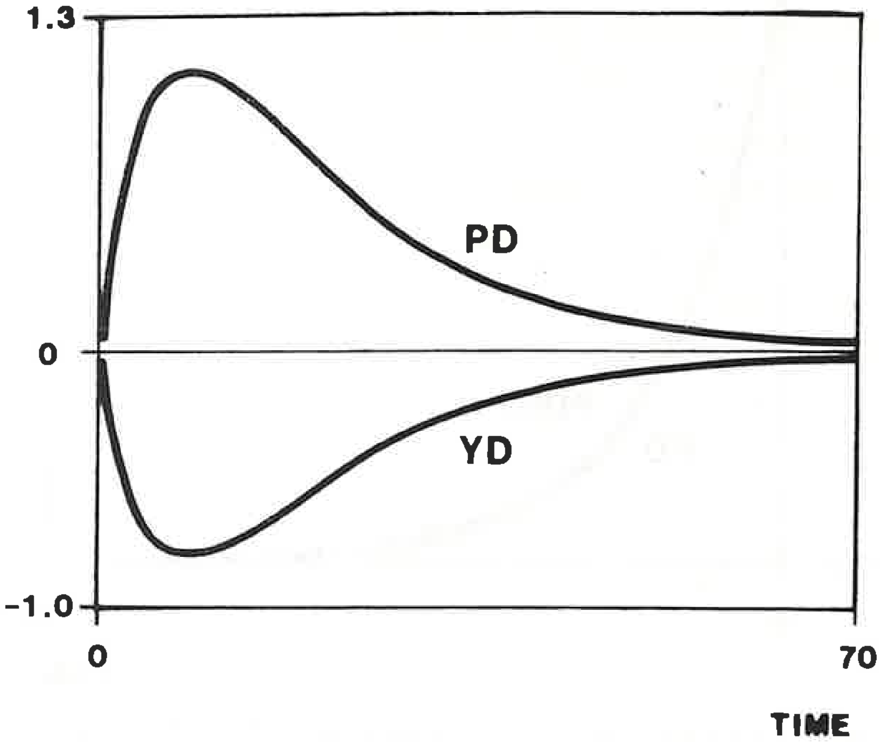
3.1.3. Numerical Examples

We provide some numerical examples which illustrate the differing effects the same exchange rate policy measures have in the case of identical and non-identical countries.

In the benchmark (identical country) case the values of the common parameters are: $a_1 = 0.3$, $a_2 = 0.2$, $a_3 = 0.1$, and $b_1 = 0.3$. In the case where $b_1 \neq b_1^*$ we assume that $b_1 = 0.4$ and $b_1 = 0.2$.

In Figure 1 the dynamic multipliers (3.37) and (3.38) are plotted.

Figure 1. Coordinated devaluation (1 %), $b_1 \neq b_1^*$.
Dynamic multipliers for p_d and y_d.



We see that each multiplier has one peak.

For comparison, the dynamic multipliers (3.14) and (3.41) are plotted in Figure 2 and the dynamic multipliers (3.15) and (3.42) in Figure 3 respectively.

Figure 2. Unilateral devaluation (1 % by country B). Dynamic multiplier for p_d. Identical and non-identical countries.

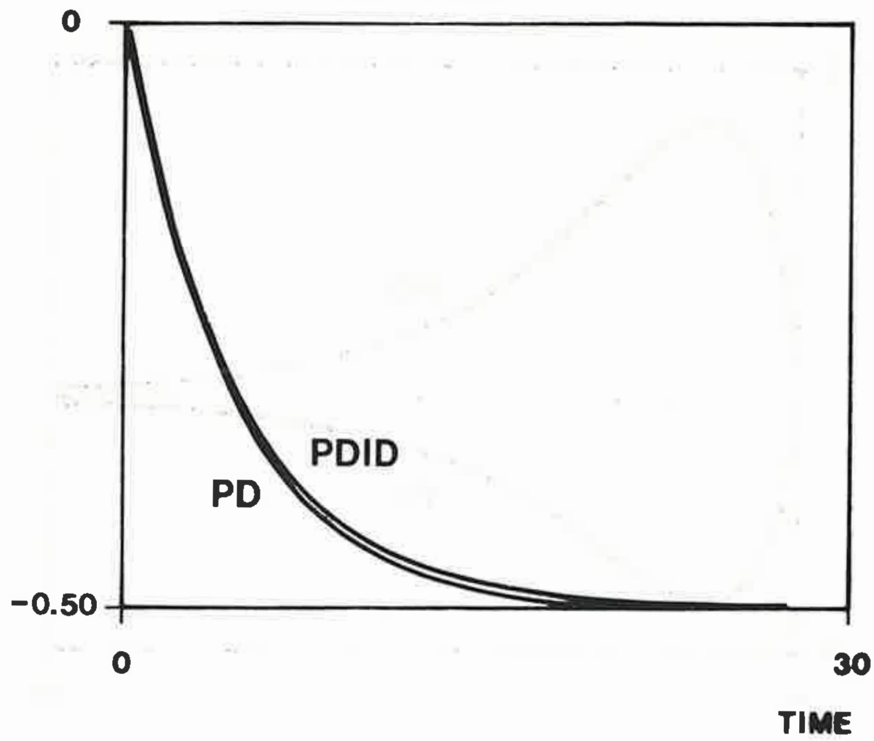
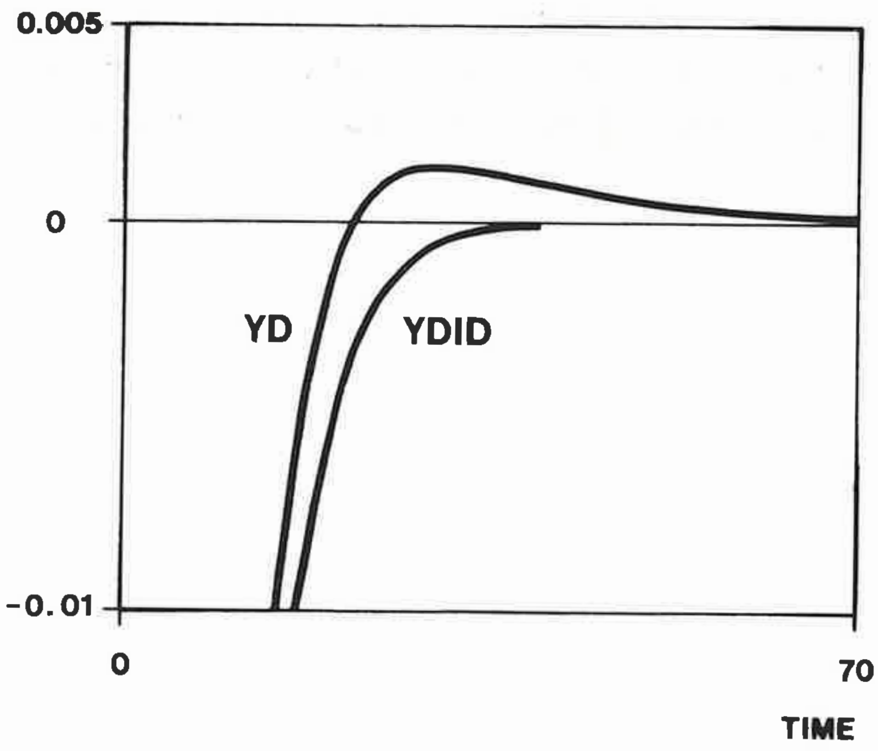


Figure 3. Unilateral devaluation (1 % by country B). Dynamic multiplier for y_d. Identical and non-identical countries.



In Figure 2, (3.14) is denoted by PDID and (3.41) by PD. An analogous notation is used in Figure 3. From Figure 2 we see that the difference $b_1 \neq b_1^*$ doesn't change very much the behaviour of p_d. Instead, as can from Figure 3, the path of y_d alters drastically. Unlike (3.15), which converges monotonically to zero, (3.42) can be positive until it becomes zero as t tends to infinity. The positive effects of the devaluation are of shorter duration when $b_1 \neq b_1^*$, $b_1 > b_1^*$. In the following chapter we analyse more thoroughly how the differences between countries A and B affect the nature of the interdependence of the exchange rate policies pursued.

4. The Interpendence of the Exchange Rate Policies of Two Small Competing Countries

4.1. Exchange Rate Policy Norms for Country A

One of our main results obtained in the previous chapter was that coordinated exchange rate policies don´t ensure that countries A and B behave in a similar way if they have different characteristics. This means that if country B undertakes a once-and-for-all devaluation, then imitating country B (i.e. undertaking immediately a devaluation by equal magnitude) is not the right reaction if country A aims at neutralizing in some sense totally the exchange rate policy measure taken by country B. What can country A then do? This is the question we deal with in the subsequent sections. We confine ourselves to analysing the reaction by country A and don´t take into account the possibility that country B can react to the reaction by country A.

When facing a devaluation undertaken by country B the authorities of country A become mainly worried about how it affects e.g. the level of employment and production through the deterioration of competitiveness. This is the major effect the authorities of country A try to neutralize.

Next, we introduce three norms which could be followed by the authorities of country A when reacting to country B´s trial to use beggar-thy-neighbour policy in improving its competitiveness.

The norms are as follows:

The output path norm: The difference between the levels of activities of countries A and B is stabilized:

(4.1) $y_d(t) = 0$

for all $t \geq 0$.[5]

The competitiveness norm: The real exchange rate between countries A and B is stabilized:

(4.2) $p_d(t) = e_d(t)$

for all $t \geq 0$.[6]

The price level norm: Price level differentials between countries A and B are stabilized:

(4.3) $p_d(t) = 0$

for all $t \geq 0$.[7]

If countries A and B were identical, then it would make no difference which norm country A follows because in that case the norms would be equivalent: following them would lead to the coordinated exchange rate policies. In the subsequent sections we conduct the analysis by assuming that the countries are not identical. It will be seen that generally the norms are not equivalent. Thus, e.g. following (4.1) can imply that (4.3) is not satisfied. However, in some cases e.g. (4.1) and (4.3) can be equivalent although the countries are not identical. It depends on the nature of differences.

Before we derive the behavioural rules corresponding to the norms (4.1)-(4.3), we consider shortly the problem of path controllability.

4.2. A Note on Path Controllability

In our model, the norms (4.1)-(4.3) are relevant for policy design only if the policy instruments e_{AC} and e_{BC} can control the paths of p^A and p^B or y^A and y^B, i.e. a given target path of p^A and p^B or y^A and y^B can be attained by using e_{AC} and e_{BC}. Therefore, some conditions for path controllability must be satisfied. (For more details, see Aoki, 1976, ch. 3 or Aoki, 1981 ch. 6).

To give an example, we analyse whether e_d can control y_d in the case of $b_1 \neq b_1^*$. Since

$$y_d(t) = -c_2 p_d(t) + c_2 e_d(t),$$

we see from the sufficient conditions presented in Aoki, 1981 (condition (i) p. 68) that if $c_2 \neq 0$, y_d is path controllable by e_b. This means that $(1 + a_3)^{-1}(2a_1 + a_2)$ must be nonzero. Because we have assumed that a_1 and a_2 are positive, this condition is satisfied.

Similarly, we can check conditions for path controllability in other cases (e.g. p_d -path controllability). It can be shown that the assumption we made earlier (a_i, a_i^* $(i = 1,2,3)$, b_1, and b_1^* are positive) ensures that the conditions required for path controllability in this study are satisfied.

4.3. Reaction Functions for Contry A

4.3.1. Output Path Norm and Competitiveness Norm, $b_1 \neq b_1^*$

We derive first the reaction functions corresponding the output path norm and the competitiveness norm.

Using Laplace transforms, (4.1) can be written in the form

$$Y_d(s) = 0.$$

By (3.36) this is equivalent to

$$-c_2P_d(s) + c_2E_d(s) = 0.$$

But this is equivalent to (4.2), the competitiveness norm. Hence, in this case the output path norm and the competitiveness norm are equivalent. By (3.34) we have

$$P_d(s) = \frac{1}{s+b_ac_2}E_d(s)+\frac{b_dc_1^2}{c_2-c_1}(\frac{1}{s+b_ac_2} - \frac{1}{s+b_ac_1})E_a(s)+\frac{b_dc_1}{s+b_ac_2}E_a(s)=E_d(s).$$

Using the definitions of e_a (E_a) and e_d (E_d), we can express this equation in the form

(4.4) $$[(b_ac_2+\frac{b_dc_1^2}{c_2-c_1}+b_dc_1)\frac{1}{s+b_ac_2}-\frac{b_dc_1^2}{c_2-c_1}\frac{1}{s+b_ac_1}-1]E_{AC}(s)$$

$$= [(b_ac_2-\frac{b_dc_1^2}{c_2-c_1} - b_dc_1)\frac{1}{s+b_ac_2} - \frac{b_dc_1^2}{c_2-c_1}\frac{1}{s+b_ac_2} - 1)]E_{BC}(s).$$

Assuming that $E_{BC}(s)$ is known, (4.4) defines an integral equation expressed in the Laplace transform form with $E_{AC}(s)$ as an unknown function. Solving (4.4) for $E_{AC}(s)$ we obtain

$$(4.5) \qquad E_{AC}(s) = \frac{s+b_a c_1 + b_d c_1}{s+b_a c_1 - b_d c_1} E_{BC}(s) = \left(1+\frac{2b_d c_1}{s+b_a c_1 - b_d c_1}\right) E_{BC}(s).$$

By inverting (4.5), $e_{AC}(t)$ can be expressed as follows (by properties 1,2,3, and 5):

$$(4.6) \qquad e_{AC}(t) = e_{BC}(t) + 2b_d c_1 \int_0^t e^{-c_1(b_a - b_d)(t-u)} e_{BC}(u)du.$$

This is the reaction function for country A given $e_{BC}(t)$. As a byproduct we have proved that if countries A and B are identical (i.e. if $b_d=0$) then $e_{AC}(t) = e_{BC}(t)$.

To get a more concrete example, we assume that country B devaluates its currency by 1 % and derive the reaction of country A by using (4.6).

Now $e_{BC} = 1$ and therefore by (4.6)

$$(4.7) \qquad e_{AC}(t) = \frac{b_a + b_d}{b_a - b_d} - \frac{2b_d}{b_a - b_d} e^{-c_1(b_a - b_d)t}.$$

We see that

$$\lim_{t \to 0} e_{AC}(t) = 1 \text{ and } \lim_{t \to \infty} e_{AC}(t) = \frac{b_a + b_d}{b_a - b_d} > 1.$$

Thus, immediately after the devaluation by country B also country A has to devaluate its currency by one percentage. Since $b_1 > b_1^*$, country A looses its competitiveness faster than country B. Therefore country A must go on devaluating in order to maintain the competitiveness. We see that it is of crucial importance that the authorities of country A realize that the countries are not identical.

We can evaluate the effects of the exchange rate policies pursued on the price levels: Because $P_d(s) = E_d(s)$, we only have to compute the path of $e_d(t)$ in order to get the path of $p_d(t)$. By (4.5) and $E_{BC}(s) = 1/s$

$$E_d(s) = \frac{b_a c_1}{b_a - b_d} \left(\frac{1}{s} - \frac{1}{s + b_a c_1 - b_d c_1}\right) = P_d(s).$$

By inverting this, $p_d(t)$ can be written as follows:

$$p_d(t) = \frac{b_a c_1}{b_a - b_d} \left(1 - e^{-c_1 (b_a - b_d) t}\right).$$

Thus if (4.1) or (4.2) is followed the rate of inflation is higher in country A then in country B.

4.3.2. Price Level Norm, $b_1 \neq b_1^*$

We assume that the authorities of country A follow the price level norm (4.3) and derive the corresponding reaction function.

It can be shown that

$$P_d(s) = 0$$

is equivalent to

(4.8) $$E_{AC}(s) = \frac{b_a^2 c_1 c_2 + (c_2 b_a - c_1 b_d)s}{b_a^2 c_1 c_2 + (c_2 b_a + c_1 d_d)s} E_{BC}(s).$$

This is the reaction function for country A when the price level norm is followed.

We assume that a once-and-for-all devaluation of 1 % is undertaken by country B and compute the reaction of country A by using (4.8). Now, by $E_{BC}(s) = 1/s$ and property 6

$$\lim_{t \to 0} e_{AC}(t) = \lim_{s \to \infty} sE_{AC}(s) = \frac{b_a c_2 - b_d c_1}{b_a c_2 + b_d c_1} < 1.$$

and

$$\lim_{t \to \infty} e_{AC}(t) = \lim_{s \to 0} sE_{AC}(s) = 1.$$

Initially, country A has to undertake a smaller devaluation. However, it has to go on devaluating until a devaluation of equal magnitude has been fulfilled.

To get the path of $e_{AC}(t)$, we invert (4.8) under the assumption $E_{BC}(s) = 1/s$ and obtain

(4.9) $$e_{AC}(t) = 1 - \left(1 - \frac{b_a c_2 - b_d c_1}{b_a c_2 + b_d c_1}\right) e^{-\frac{b_a^2 c_1 c_2}{b_a c_2 + b_d c_1} t}.$$

This implies that the convergence is monotonic.

The effects of the exchange rate policies pursued on $y_d(t)$ can be computed as follows: By (3.36)

(4.10) $$Y_d(s) = c_2 E_d(s).$$

Using (4.8) and $E_{BC}(s) = 1/s$ when forming $E_d(s)$, and inverting this, $e_d(t)$ can be written in the form

$$e_d(t) = -\frac{2b_d c_1}{b_a c_2 + b_d c_1} e^{-\frac{b_a^2 c_1 c_2}{b_a c_2 + b_d c_1} t}.$$

Hence by (4.10)

$$y_d(t) = -\frac{2b_d c_1 c_2}{b_a c_2 + b_d c_1} e^{-\frac{b_a^2 c_1 c_2}{b_a c_2 + b_d c_1} t},$$

which is negative for finite positive values of t. Thus in the intermediate run activity is at a higher level in country B than in country A if the price level norm is followed by country A.

We next construct another set of examples by assuming that the only difference between the characteristics of countries A and B is: $a_2 \neq a_2^*$, $a_2 > a_2^*$, i.e. the good produced in country A is a closer substitute for the good produced in country C than that produced in country B.

4.3.3. Output Path Norm and Price Level Norm, $a_2 \neq a_2^*$

Assuming that $a_2 \neq a_2^*$, $a_2 > a_2^*$, is the only difference between countries A and B the model can be written in the form

$$(4.11) \quad \begin{pmatrix} y_a \\ y_d \end{pmatrix} = \begin{pmatrix} (1-a_3)^{-1} & 0 \\ 0 & (1+a_3)^{-1} \end{pmatrix} \left[\begin{pmatrix} -a_{2a} & -a_{2d} \\ -a_{2d} & -2a_1 - a_{2a} \end{pmatrix} \begin{pmatrix} p_a \\ p_d \end{pmatrix} + \begin{pmatrix} a_{2a} & a_{2d} \\ a_{2d} & 2a_1 + a_{2a} \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix} \right],$$

where $a_{2a} = (a_2 + a_2^*)/2$ and $a_{2d} = (a_2 - a_2^*)/2$,

$$(4.12)\qquad \begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ 0 & b_1 \end{pmatrix} \begin{pmatrix} y_a \\ y_d \end{pmatrix}.$$

Inserting (4.11) into (4.12), (4.12) is replaced by

$$\begin{pmatrix} \dot{p}_a \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} -b_1c_{1a} & -b_1c_{3d} \\ -b_1c_{4d} & -b_1c_{2a} \end{pmatrix} \begin{pmatrix} p_a \\ p_d \end{pmatrix} + \begin{pmatrix} b_1c_{1a} & b_1c_{3d} \\ b_1d_{4d} & b_1c_{2a} \end{pmatrix} \begin{pmatrix} e_a \\ e_d \end{pmatrix},$$

where $c_{1a} = (1-a_3)^{-1}a_{2a}$, $c_{2a} = (1+a_3)^{-1}(2a_1+a_{2a})$,

$c_{3d} = (1-a_3)^{-1}a_{2d}$ and $c_{4d} = (1+a_3)^{-1}a_{2d}$.

Now we substitute c_{1a} and c_{2a} for c_1 and c_2 in (3.8) and define this and (3.9) as our benchmark model in the case where $a_2 \neq a_2^*$, $a_2 > a_2^*$. Then proceeding in the same way as above, we obtain the following set of equations expressed by Laplace transforms:

$$(4.13)\qquad P_a(s) = \frac{b_1c_{1a}}{s+b_1c_{1a}}E_a(s) - \frac{b_1c_{2a}c_{3d}}{(s+b_1c_{1a})(s+b_1c_{2a})}E_d(s) + $$
$$+ \frac{b_1c_{3d}}{s+b_1c_{1a}}E_d(s),$$

$$(4.14)\qquad P_d(s) = \frac{b_1c_{2a}}{s+b_1c_{2a}}E_d(s) - \frac{b_1^2c_{1a}c_{4d}}{(s+b_1c_{1a})(s+b_1c_{2a})}E_a(s) + $$
$$+ \frac{b_1c_{4d}}{s+b_1c_{2a}}E_a(s),$$

$$(4.15)\qquad Y_a(s) = -c_{1a}P_a(s) - c_{3d}P_d(s) + c_{1a}E_a(s) + $$

$$+ c_{3d}E_d(s),$$

(4.16) $$Y_d(s) = -c_{4d}P_a(s) - c_{2d}P_d(s) + c_{4d}E_a(s) + c_{2a}E_d(s).$$

Using these equations, we derive the reaction functions compatible with (4.1) - (4.3).

First we prove that in this case the output path norm and the price level norm are equivalent: Taking the Laplace transforms from

$$\dot{p}_d(t) = b_1 y_d(t)$$

and assuming that $p_d(0) = 0$ we have by property 4

$$sP_d(s) = b_1 Y_d(s).$$

From this we see that (4.1) is equivalent to (4.3).

Now it can be shown that

$$Y_d(s) = 0$$

is equivalent to

(4.17) $$E_{AC}(s) = \frac{b_1c_{1a}c_{2a} - b_1c_{3d}c_{4d} + (c_{2a} - c_{4d})s}{b_1c_{1a}c_{2a} - b_1c_{3d}c_{4d} + (c_{2a} + c_{4d})s} E_{BC}(s),$$

which is the reaction function for country A when the output path norm or the price level norm is followed.

Inverting (4.17) under the assumption $E_{BC}(s) = 1/s$, we obtain

$$e_{AC}(t) = 1 - \left(1 - \frac{c_{2a} - c_{4d}}{c_{2a} + c_{4d}}\right) e^{\frac{b_1(c_{1a}c_{2a} - c_{3d}c_{4d})}{c_{2a} + c_{4d}} t} .$$

Taking the limits, we have

$$\lim_{t \to 0} e_{AC}(t) = \frac{c_{2a} - c_{4d}}{c_{2a} + c_{4d}} \text{ and } \lim_{t \to \infty} e_{AC}(t) = 1.$$

Hence, immediately after the devaluation by country B country A has to undertake a smaller devaluation. However, thereafter country A has to go on devaluating until a devaluation of equal magnitude has been fulfilled (as t has tended to infinity).

4.3.4. Competitiveness norm, $a_2 \neq a_2^*$

We see from (4.16) that in this case the competitiveness norms is not equivalent to the output path norm.

It can be shown that now

$$P_d(s) = E_d(s)$$

is equivalent to

$$E_{AC}(s) = \frac{b_1c_{1a} + b_1c_{4d} + s}{b_1c_{1a} - b_1c_{4d} + s} E_{BC}(s) = \left(1 + \frac{2b_1c_{4d}}{b_1c_{1a} - b_1c_{4d} + s}\right) E$$

Inverting this, we have by properties 1,2,3, and 5

$$(4.18) \qquad e_{AC}(t) = e_{BC}(t) + 2b_1c_{4d}\int_0^t e^{-(b_1c_{1a}-b_1c_{4d})(t-u)}e_{BC}(u)du$$

which is the reaction function for country A when the competitiveness norm is followed. The reaction to a once-and-for-all devaluation of 1 % by country B is by (4.18) expressed as follows

$$e_{AC}(t) = \frac{c_{1a} + c_{4d}}{c_{1a} - c_{4d}} + (1 - \frac{c_{1a} + c_{4d}}{c_{1a} - c_{4d}}) e^{-b_1(c_{1a} - c_{4d})t}.$$

Now

$$\lim_{t\to 0} e_{AC}(t) = 1 \text{ and } \lim_{t\to\infty} e_{AC}(t) = \frac{c_{1a} + c_{4d}}{c_{1a} - c_{4d}} > 1.$$

Immediately after the devaluation by country B, a devaluation of equal magnitude must be undertaken. Because $a_2 > a_2^*$, activity rises higher in country A than in country B. If nothing else is done, the competitiveness of country A will deteriorate due to the higher rate of inflation. For that reason additional devaluations are required until in the long run $\lim_{t\to\infty} e_{AC}(t) = (c_{1a}+c_{4d})/(c_{1a}-c_{4d})$.

We have seen that in the case where $a_2 \neq a_2^*$ the output path norm and the price level norm are equivalent. Therefore it makes no difference which one the authorities of country A follow. However, the competitiveness norm is now not equivalent to the output path norm.

We have used a model as simple as possible in order to be able to describe the basic ideas in a very simple form. In Appendix B a slightly modified model is used in analysing the problem of interdependence. The purpose of the appendix is to illustrate how adding the dimensionality of the dynamics affects the analysis.

5. Concluding Comments

In this essay, we have introduced a framework by which the interdependence of the exchange rate policies of two small countries (our home country and our neighbouring country) competing with each other can be studied. We combined Aoki´s method for analysing interdependent economies and the ideas on exchange rate policy rules put forward especially by Korkman.

One of our main objectives was to analyse how structural differences between two countries affect the nature of the interdependence of the exchange rate policies pursued. We saw that, if the two countries are not identical, coordinated exchange rate policies do not ensure that the countries behave in a similar way. Therefore, if the authorities of our home country know that the structures of these countries differ, they should realize that just imitating the policy of the competitor is probably false policy, if they are trying to neutralize in some sense totally the effects of the exchange rate measures taken by the competitor. However, they should also realize that if the countries differ they can´t neutralize all consequences of the exchange rate policy pursued by the competitor if they only have the exchange rate policy instrument at their disposal: e.g. stabilizing the real exchange rate between the countries can give rise to a higher rate of inflation in our home country if it is more sensitive to changes in activity.

We saw that if the differences between the countries are reflected in differences in the dynamics of these countries, continuing changes in exchange rates are required to neutralize a once-and-for-all change by the competitor if e.g. stabilizing the real exchange rate is aimed at. This shows how difficult it is in practice to try to achieve the targets we regarded as norms.

We have seen that structural differences should somehow be taken into account when reactions to the exchange rate policy measures taken by our competitor are planned and implemented. So far this fact has been (completely) neglected when discussing in Finland the exchange rate policies pursued by Finland and Sweden.

Notes

1 The fixed exchange rate rule means that an index of prices of foreign exchange is stabilized; the inflation norm means that the domestic currency value of a price index of foreign goods is stabilized, and the competitiveness norm means that the real exchange rate is stabilized.

2 Because $\bar{Y}^C$ and $\bar{P}^C$ are assumed to be fixed, these variables drop out.

3 Under the assumption $p_d(0) = 0$, a non-negative $p_d(t)$ implies that the rates of inflation differ in the countries.

4 There are at least two good reasons for using the method of Laplace transforms: It is a very efficient method for solving linear differential and integral equations, and at the same time it is also a beautiful way of solving these equations.

5 This norm was used by Aoki, when he analysed the interdependence of the monetary policies of two countries. See Aoki, 1981, pp. 198-200.

6 This is analogous to the competitiveness norm used by Korkman.

7 As noted earlier, (4.3) also means that the difference between the rates of inflation is stabilized.

References

Aoki, M. (1976): Optimal Control and Systems Theory in Dynamic Economic Analysis. New York.

Aoki, M. (1981): Dynamic Analysis of Open Economies. New York.

Boyce, W.E. - DiPrima, R.C. (1965): Elementary Differential Equations and Boundary Value Problems. New York.

Korkman, S. (1980): Exchange Rate Policy, Employment and External Balance. Helsinki.

Appendix A Laplace Transforms[1]

The basic idea in the use of Laplace transforms is that one can solve e.g. linear differential and integral equations by solving algebraic equations.

Definition: Suppose that f(t) is defined for all $t \geq 0$. The Laplace transform of f, which is denoted by F(s) or $L(f(t))$ is defined by the equation

$$L(f(t)) = F(s) = \int_0^{\infty} e^{-st} f(t)dt,$$

whenever this integral converges.

In general s can be complex, but we consider only real values of s.

Property 1. Let $f(t) = a$, $t \geq 0$. Then

$$L(a) = a/s, \; s > 0.$$

Property 2. Let $F(s) = L(f(t))$. Then

$$L(e^{at} f(t)) = F(s-a).$$

Property 3 (linearity). Assume that Laplace transforms for functions f_1 and f_2 exist. Then

$$L(c_1 f_1 + c_2 f_2) = c_1 L(f_1) + c_2 L(f_2),$$

where c_1 and c_2 are arbitrary constants.

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1 This appendix is based on Boyce-DiPrima(1965) and Aoki (1976).

Property 4. Suppose that Laplace transform for f(t) exists. Then

$$L(f'(t)) = sL(f(t)) - f(0).$$

This property is used when differential equations are solved.

Property 5 (Laplace transforms for convolution integrals). Assume $F(s) = L(f(t))$ and $G(s) = L(g(t))$ both exist for $s \geq a > 0$. Then

$$H(s) = F(s)G(S) = L(h(t)), \quad s > a,$$

where

$$h(t) = \int_0^t f(t-u)g(u)du = \int_0^t f(u)g(t-u)du$$

is the convolution of f and g.

Property 6. Under some mild regularity conditions on f, we have

$$\lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t),$$

which is called as Initial Value Theorem, and

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t),$$

which is called as Final Value Theorem.

We use these properties both when taking Laplace transforms and when inverting functions which are in the Laplace transform form. The inverse transform of F(s) is denoted by $f(t) = L^{-1}(F(s))$.

Appendix B A Slightly Modified Model

In this appendix, we construct a slightly modified model: we add into the model (2.1)-(2.4) two dynamic equations. The variational form of the model can be written as follows:

(1) $$y^{DA} = a_1(\, p^B+e_{AC}-e_{BC}-p^A \,)+a_2(e_{AC}-p^A)$$
$$+b_1y^A+b_2y^B \;,$$

(2) $$y^{DB} = a_1^*(\, p^A+e_{BC}-e_{AC}-p^B \,)+a_2^*(e_{BC}-p^B)$$
$$+b_1^*y^B+b_2^*y^A \;,$$

(3) $$\dot{y}^B = c_1(\, y^{DA}-y^A \,),$$

(4) $$\dot{y}^B = c_1^*(\, y^{DB}-y^B \,),$$

(5) $$\dot{p}^A = c_2(\, y^{DA}-y^A \,),$$

(6) $$\dot{p}^B = c_2^*(\, y^{DB}-y^B \,).$$

Now (1) and (2) are the aggregate demand functions for countries A and B. By (3) and (4) outputs in countries A and B adjust in proportion to excess demands (Lundberg´s lag in production). Equations (5) and (6) describe the price dynamics of countries A and B.

Next, we illustrate how we can derive reaction functions for country A when (1)-(6) are used.

We assume first that countries A and B are identical. Then by using averages and differences the model can be expressed in the following form:

$$(9)\quad \begin{pmatrix} \dot{z}_a \\ \dot{z}_d \end{pmatrix} = \begin{pmatrix} G_a & 0 \\ 0 & G_d \end{pmatrix} \begin{pmatrix} z_a \\ z_d \end{pmatrix} + \begin{pmatrix} H_a \\ 0 \end{pmatrix} e_a + \begin{pmatrix} 0 \\ H_d \end{pmatrix} e_d,$$

where $z_a = (y_a, p_a)'$, $z_d = (y_d, p_d)'$,

$$G_a = \begin{pmatrix} -c_1(1-b_1)+c_1b_2 & -c_1a_2 \\ -c_2(1-b_1)+c_2b_2 & -c_2a_2 \end{pmatrix},$$

$$G_d = \begin{pmatrix} -c_1(1-b_1)-c_1b_2 & -2c_1a_1-c_1a_2 \\ -c_2(1-b_1)-c_2b_2 & -2c_2a_1-c_2a_2 \end{pmatrix},$$

$$H_a = \begin{pmatrix} c_1a_2 \\ c_2a_2 \end{pmatrix}, \quad H_d = \begin{pmatrix} 2c_1a_1+c_1a_2 \\ 2c_2a_1+c_2a_2 \end{pmatrix}.$$

We see that the average dynamics separates from the difference dynamics.

Now we assume that $c_1 \neq c_1^*$, $c_1 > c_1^*$, and derive the reaction function compatible with the output path norm. We choose (9), where $c_{1a} = (c_1+c_1^*)/2$ has been substituted for c_1, as our benchmark model. Denoting the solution of the benchmark model by superscript zero, we obtain by using Laplace transforms

$$sZ_a^o(s) - z_a(0) = G_aZ_a^o(s) + H_aE_a(s),$$

$$sZ_d^o(s) - z_d(0) = G_dZ_d^o(s) + H_dE_d(s).$$

Solving these for $Z_a^o(s)$ and $Z_d^o(s)$ under the assumption $z_a(0) = z_d(0) = 0$ we have

(10) $$Z_a^o(s) = (sI - G_a)^{-1}H_aE_a(s),$$

(11) $$Z_d^o(s) = (sI - G_d)^{-1}H_dE_d(s).$$

The perturbed model can be expressed as follows:

(12) $$\begin{pmatrix} \dot{z}_a \\ \dot{z}_d \end{pmatrix} = \begin{pmatrix} G_a & G_{ad} \\ G_{da} & G_d \end{pmatrix}\begin{pmatrix} z_a \\ z_d \end{pmatrix} + \begin{pmatrix} H_a \\ H_{da} \end{pmatrix} e_a +$$

$$\begin{pmatrix} H_{ad} \\ H_d \end{pmatrix} e_d \ ,$$

where

$$G_{ad} = \begin{pmatrix} -c_{1d}(1-b_1+b_2) & -c_{1d}(2a_1+a_2) \\ 0 & 0 \end{pmatrix},$$

$$G_{da} = \begin{pmatrix} -c_{1d}(1-b_1-b_2) & -c_{1d} \\ 0 & 0 \end{pmatrix},$$

$$H_{da} = \begin{pmatrix} c_{1d}a_2 \\ 0 \end{pmatrix} \ , \ H_{ad} = \begin{pmatrix} c_{1d}(2a_1+a_2) \\ 0 \end{pmatrix},$$

and $c_{1d} = (c_1 - c_1^*)/2$.

Using (9) and (12) we can derive the solutions for $\delta Z_a(s)$ and $\delta Z_d(s)$, $\delta Z_a = Z_a - Z_a^o$, $\delta Z_d - Z_d^o$. They are:

(13) $$\delta Z_a(s) = (sI - G_a)^{-1}H_{ad}Z_d^o(s) +$$

$$(sI - G_a)^{-1}H_{ad}E_d(s),$$

(14) $$\delta Z_d(s) = (sI - G_d)^{-1} H_{da} Z_a^o(s) +$$

$$+ (sI - G_d)^{-1} H_{da} E_a(s).$$

Substituting (10)-(11) into (13)-(14) we obtain:

(15) $$Z_a(s) = Z_a^o(s) + \delta Z_a(s) = (sI - G_a)^{-1} H_a E_a(s) +$$

$$+ (sI - G_a)^{-1} H_{ad} (sI - G_d)^{-1} H_d E_d(s) +$$

$$+ (sI - G_a)^{-1} H_{ad} E_d(s)$$

(16) $$Z_d(s) = Z_d^o(s) + \delta Z_d(s) = (sI - G_d)^{-1} H_d E_d(s) +$$

$$(sI - G_d)^{-1} H_{da} (sI - G_a)^{-1} H_a E_a(s) +$$

$$(sI - G_d)^{-1} H_{da} E_a(s).$$

Using (15) and (16) we could repeat the analysis we conducted by (3.33)-(3.36). We, however, only derive the reaction function corresponding to the output path norm.

Picking up $Y_d(s)$ from (16), we can show that

$$Y_d(s) = 0$$

is equivalent to

(17) $$E_{AC}(s) = \left(1 - \frac{2c_{1d}a_2 s(s+2c_2a_1+c_2a_2)}{q_1(s)c_{1a}(2a_1+a_2)+c_{1d}a_2 s(s+2c_2a_1+c_2a_2)}\right)$$

where $q_1(s) = (s+s_1)(s+s_2)$ and s_1, s_2 are the negatives of the distinct eigenvalues of the matrix G_a $(s_2>s_1$

by assumption). This is the reaction function for country A when $c_1 \neq c_1^*$ and the output path norm is followed.

To get a concrete example, we assume that country B devaluates its currency by 1 % i.e. we assume that $E_{BC}(s) = 1/s$. By property 6

$$\lim_{t \to 0} e_{AC}(t) = \lim_{s \to \infty} sE_{AC}(t) = \frac{c_{1a}(2a_1+a_2)-c_{1d}a_2}{c_{1a}(2a_1+a_2)+c_{1d}a_2} < 1,$$

and

$$\lim_{t \to \infty} e_{AC}(t) = \lim_{s \to 0} sE_{AC}(s) = 1.$$

Hence immediately after the devaluation by country B also country A has to devaluate its currency but by a smaller amount. (This is due to the fact that output adjusts faster in country A than in country B.) Country A must, however, go on changing the value of its currency until a devaluation of equal magnitude has been undertaken.

Obtaining the path of e_{AC} is now not so easy as in the case of the simpler model. We simplify the problem by following Aoki (see Aoki, 1981, p.220 footnote 26) and approximate (17) by

$$(18) \qquad E_{AC}(s) = \left(1 - \frac{2c_{1d}a_2 s(s+2c_2a_1+c_2a_2)}{q_1(s)c_{1a}(2a_1+a_2)}\right) E_{BC}(s).$$

Assuming that $E_{BC}(s) = 1/s$ and inverting (18), the path of e_{AC} can be written as follows:

$$(19) \qquad e_{AC}(t) = 1 - \frac{2c_{1d}a_2}{c_{1a}(2a_1+a_2)(s_2-s_1)}\Big((-s_1+2c_2a_1+c_2a_2)e^{-s_1t} - (-s_2+2c_2a_1+c_2a_2)e^{-s_2t}\Big).$$

As can be seen from (19), by using (18) we can describe the behaviour of e_{AC} by making use of the eigenvalues associated with the benchmark model. Furthermore, unlike in the case of the simpler model, (19) does not necessarily behave monotonically with time. Thus adding dimensionality can affect remarkably the nature of reaction functions.